

DESIGN & ANALYSIS OF ALGORITHMS (CSE2201)

Time Allotted : 2½ hrs

Full Marks : 60

Figures out of the right margin indicate full marks.

*Candidates are required to answer Group A and
any 4 (four) from Group B to E, taking one from each group.*

Candidates are required to give answer in their own words as far as practicable.

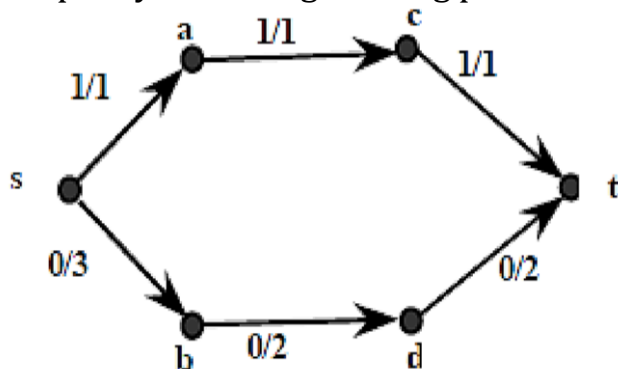
Group – A

1. Answer any twelve:

12 × 1 = 12

Choose the correct alternative for the following

- (i) The function $f(n) = 5n^2 + 100$ is
 (a) $\Theta(n^2)$ (b) $\Omega(n^2)$
 (c) $O(n^2)$ (d) all of the above
- (ii) To sort 4 elements, we need ____ comparisons in the worst case.
 (a) 8 (b) 5 (c) 16 (d) 6
- (iii) Let P be a Quick Sort program applied to the two I/P sequences: I1: [1, 2, 3, 4, 5] and I2: [5, 4, 3, 2, 1], where the last element is always used as the pivot. t1 and t2 are the number of comparisons P requires to sort the I1 and I2 respectively. Then which one is correct?
 (a) $t_1 = t_2$ (b) $t_1 \leq t_2$
 (c) $t_1 \geq t_2$ (d) None of these
- (iv) Which of the following statements is true?
 (a) If one vertex v is not reachable from source s of Graph G (V, E), the weight of the shortest path from s to v is zero.
 (b) Bellman-Ford algorithm can find the shortest paths to all vertices even if there is a negative weight cycle reachable from source s in a graph G (V, E).
 (c) Kruskal's algorithm does not check whether a newly selected edge creates a cycle with already selected edges.
 (d) Output of Prim's algorithm is the predecessor subgraph of the graph G (V, E).
- (v) Let G be an undirected connected graph with distinct edge weights, let e' be the edge with maximum weight and e'' be the edge with minimum weight. Which of the following statement(s) is/are FALSE?
 (a) Every minimum spanning tree of G contains e'' (b) No minimum spanning tree contains e'
 (c) G has a unique minimum-spanning tree (d) None of the above
- (vi) We know that the Greedy techniques may produce sub-optimal results in some cases. Among the below-mentioned greedy algorithms which one does not produce sub-optimal results?
 (a) Fractional Knapsack (b) Kruskal Algorithm
 (c) (a) & (b) both (d) None of them
- (vii) The complexity of the Floyd Warshall algorithm using the Adjacency Matrix is -
 (a) $O(n^2 \lg n)$ (b) $O(n^3)$
 (c) $O(n^2)$ (d) $O(n^4)$
- (viii) If we use a biased coin whose probability of getting a Head is 4/5 [$P(H) = 4/5$ and $P(T) = 1/5$], then the expected size (total number of elements present) of the Skip List is
 (a) $5/4 n$ (b) $4n$ (c) $5n$ (d) $6n$
- (ix) Which of the following is NP-complete?
 (a) Travelling salesman problem, longest simple path problem
 (b) Matrix-chain multiplication problem, Fraction knapsack problem
 (c) Edge-cover problem, 2-CNF
 (d) Finding all pairs shortest path of a graph with negative edges
- (x) The residual capacity of the augmenting path for the flow network shown below is



- (a) 0.5 (b) 1 (c) 1.5 (d) 2

- (xi) A simple acyclic path between source and sink through a residual flow network that passes through only positive-weight edges is called an _____.
- (xii) The time complexity of the Bellman-Ford algorithm for a directed, weighted graph of $|V|$ vertices and $|E|$ edges is _____.
- (xiii) NP is the class of languages that can be _____ by a polynomial time algorithm.
- (xiv) The worst-case time complexity for binary search is _____ when applied to an array of size n .
- (xv) All problems of _____ are in NP but problems of NP are not in _____.

Group - B

2. (a) Write the pseudo code of the procedure MERGE(A, p, q, r), where A is an array and p, q, r are indices such that $p \leq q < r$. The procedure should merge the two sorted sub-arrays $A[p..q]$ and $A[q+1..r]$ into a single sorted array. [[CO2](Understand, Remember/LOCQ)]
 (b) Apply the above procedure to merge the two sorted sub-arrays $[3, 8, 10, 12]$ and $[1, 5, 7]$ into a single sorted array. Show all the steps. [[CO2](Apply/IOCQ)]
 (c) Analyze the worst-case time complexity of the MERGE procedure written in part (a). [[CO4](Apply/IOCQ)]

$5 + 4 + 3 = 12$
3. (a) Prove that the average-case time complexity of Quick Sort is $O(n \log n)$ using the recurrence relation approach. [[CO4](Apply/IOCQ)]
 (b) Illustrate how the partition function of Quick Sort works on the array $[19, 8, 5, 12, 8, 7, 4, 21]$. Choose the first element of the array as a pivot element. [[CO2](Apply/IOCQ)]

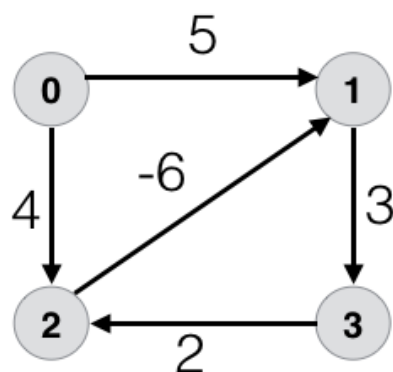
$6 + 6 = 12$

Group - C

4. (a) Fill up the spaces (Asymptotic values are fine) in the following table with respect to Prim's algorithm on the graph $G(V, E)$.

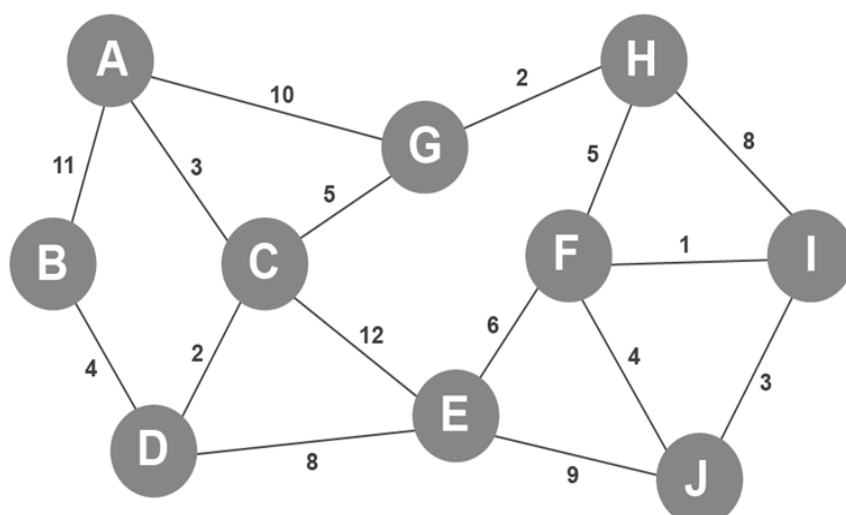
	How the Priority Queue will be implemented		
	Array	Binary Heap	Fibonacci Heap
Each Extract-Min will take			
Each Decrease-key will take			
Final Asymptotic Complexity			

[[CO4](Analyze/LOCQ)]
 (b) The fractional knapsack problem can be solved using a greedy algorithm. Is the above statement true? Justify your answer. [[CO2](Understand/IOCQ)]
 (c) Show the stepwise execution of the Bellman-Ford algorithm on the following graph.



- (d) Why do you think the greedy strategy is applied in the Bellman-Ford algorithm? [[CO3](Apply/IOCQ)]
[[CO2](Understand/IOCQ)]

$4.5 + 2.5 + 3 + 2 = 12$
5. (a) Write two components of the Greedy strategy observed in the Fractional Knapsack problem. [[CO1](Remember/LOCQ)]
 (b) Find the minimum spanning tree of the following graph using Kruskal's algorithm. Show each step.

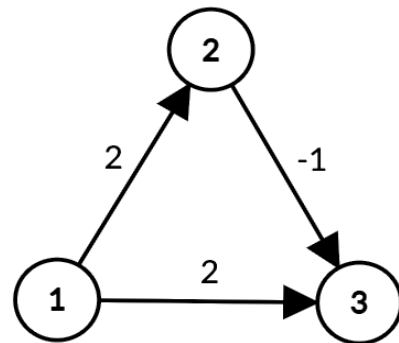


- (c) How do aggregate analysis and average case analysis differ? [[CO3](Apply/IOCQ)]
[[CO2](Understand/LOCQ)]

$4 + 6 + 2 = 12$

Group - D

6. (a) Matrices A, B, and C, of dimensions 10×24 , 24×6 , and 6×17 respectively, are provided. You are told to ensure that when computing the matrix product A.B.C. the number of scalar multiplications will be a minimum. How would you parenthesize the product, and what would be the minimum number of scalar multiplications you would need to perform? [[CO3](Apply/IOCQ)]
- (b) Show each step of the execution of the Floyd-Warshall algorithm on the following graph.



- (c) Describe the recursive definition for solving the 0/1 Knapsack problem.

[[CO3](Apply/IOCQ)]
[[CO2](Understand/LOCQ)]
4 + 4 + 4 = 12

7. (a) Consider the usual algorithm for matrix multiplication and consider the matrix chain product $M = M_1 \times M_2 \times M_3 \times M_4$ where the dimensions of the matrices are 10×20 , 20×30 , 30×40 , and 40×50 respectively. Assuming that the multiplication of a $p \times q$ matrix with a $q \times r$ matrix requires pqr operations, compute the minimum number of operations required to evaluate the product, using dynamic programming, showing key steps. [[CO2, CO3](Apply/IOCQ)]
- (b) Give the outline of an $O(n^2)$ dynamic programming algorithm that computes $T(n)$ defined as follows

$$T(n) = 2 \sum_{i=1}^{n-1} T(i) * T(i-1)$$

$$T(0) = T(1) = 2$$

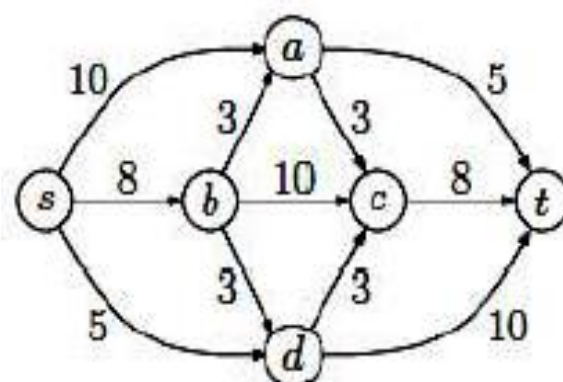
[[CO2, CO3](Apply/HOCQ)]
6 + 6 = 12

Group - E

8. (a) Use the following example to show that the Clique Decision Problem is NP-hard by using the result that the 3-CNF-SAT problem is computationally hard. Reduce the CNF, $(x \vee y \vee z') \wedge (x \vee y' \vee w') \wedge (x' \vee z \vee w') \wedge (z' \vee y' \vee w')$ to an instance of a clique decision problem, where x, y, z , and w are Boolean variables. What is the size of the clique that you will be looking for? Please write all the steps and the terms used in your steps in detail. [[CO3, CO5](Analyse/IOCQ)]
- (b) Once you have proved a problem to be NP-complete, just mention at least three options that are available to you if you still need to solve the problem. [[CO1, CO2](Remember, Understand/LOCQ)]
- (c) Fill in the six blanks with word(s) or symbols wherever appropriate to make perfect sense and then write the full text in your copy –
- NP is the class of languages that can be _____ by a _____ time algorithm. More precisely a language L belongs to NP if there exists a two-input polynomial time algorithm A and a _____ c such that $L = \{x \in \{0, 1\}^* : \text{there exists a certificate } y \text{ with } |y| = O(|x|^c) \text{ such that } A(x, y) = 1.\}$ [[CO5](Apply/IOCQ)]

6 + 3 + (0.5 × 6) = 12

9. (a) Apply the FORD-FULKERSON algorithm on the following flow network to find the maximum flow in the network. s & t denotes source & destination and the weights associated with every edge represent the capacity of the respective edge.



- (b) Prove that using the linked-list representation of disjoint sets and the weighted-union heuristic, a sequence of m MAKE-SET, UNION, and FIND-SET operations, n of which are MAKE-SET operations, takes $O(m + n \lg n)$ time. [[CO2, CO3](Apply/IOCQ)]

[[CO4](Analyse/IOCQ)]
7 + 5 = 12

Cognition Level	LOCQ	IOCQ	HOCQ
Percentage distribution	23.45	67.18	9.37

