M.TECH/CSE/1ST SEM /CSEN 5102/2015
2015

Algorithms & Complexity
(CSEN 5102) 22

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the best alternative for the following: **10 x 1=10**

(i) The best-case time complexity for binary search from an array of n sorted elements is
(a) O(n) (b) O(log n) (c) O(n/log n) (d) O(1).

(ii)
```
for i = 1 to m
{
  for j = 1 to i
     sum = sum + A [ i ][ j ];
}
```
The time complexity of the above code snippet in worst case is
(a) $O(m^2)$ (b) $O(m^3)$ (c) O(m) (d) None of these.

(iii) The Single-source shortest path problem in a graph G becomes undefined if there exists _____from the source.
(a) a –ve weight cycle in the graph
(b) a –ve edge reachable from the source
(c) multiple –ve edges in the graph
(d) a –ve weight cycle reachable from the source.

(iv) In the KMP algorithm for pattern matching, the suffix function σ(x) is the largest ___ of the pattern P that is also a _____ of x.
(a) prefix, suffix (b) prefix, prefix
(c) suffix, prefix (d) suffix, suffix.

(v) Worst case time complexity for inserting an element in a sorted array so that it stays sorted is
(a) O(1) (b) O(n) (c) $O(n^2)$ (d) None of these.

(vi) The solution of the recurrence relation is $T(n) = 2\,T(\lfloor n/2 \rfloor) + n$
(a) $O(n \log_2 n)$ (b) $\Omega(n \log_2 n)$
(c) $\Theta(n \log_2 n)$. (d) None of the above.

(vii) The number of edges in a DFS forest (having 9 connected components) of a graph with 100 vertices is

(a) 99 (b) 91 (c) 90 (d) Less than 90.

(viii) Which of the following basic algorithms can be used to most efficiently determine the presence of a cycle in a given graph?

(a) MST algorithm (b) Ford-Fulkerson's algorithm
(c) BFS algorithm (d) DFS algorithm.

(ix) Finding an Eulerian Circuit in a given graph is

(a) NP-hard (b) NP-complete
(c) in P (d) None of the above.

(x) A student proved that the longest path problem is NP-complete by reducing it to another already known NP-complete problem named set-cover problem. His teacher said the proof is not correct and did not give him any marks because

(a) the teacher does not understand NP-completeness as it is a difficult chapter.
(b) the student got a wrong answer about the hardness of longest path problem.
(c) the method of proof given by the student was wrong.
(d) none of the above is true.

Group – B

2.(a) Let g(n) be a function of n. Define the following asymptotic notations with example diagrams – O(g(n)), Ω (g(n)) and Θ(g(n)).

(b) A sequence of n operations is performed on a data structure. The cost of ith operation is C(i) = i^2, if i is an exact power of 3
= 3, otherwise.
Calculate the exact expression for cost for n successive operations. Use Aggregate Analysis to determine the amortized cost per operation.

(2 + 2 + 1) + 7 = 12

3.(a) Let's assume that a polynomial of degree m is represented as $t(n) = \sum_{i=0}^{m} a_i n^i$. Prove that $t(n) = O(n^m)$.

(b) Write a recursive algorithm to solve Towers of Hanoi problem and then analyze your algorithm in worst case.

4 + (4+4) = 12

Group – C

4.(a) What is the main drawback of bubblesort algorithm that remains even if we use an improved version of it? What is main strategy used by cocktail-shaker sort that makes it different from bubblesort? What is the main achievement of cocktail-shaker sort? Give a small example using less than 10 elements to illustrate it. Does any other popular sorting method use that achievement as a part of its strategy?

(c) Show that the 2nd smallest of **n** elements can be found with $n + \lceil \lg n \rceil - 2$ comparisons in the worst case.

(1 + 1+ 4) + 6 = 12

5.(a) You have an array of size 20 in which 16 numbers are arranged into a max-heap. If you want to add a new number into the heap, how will you do that? You may either give the pseudo-code or explain it in plain English.

(b) You are given a sorted array of n distinct integers. When does the worst case happen in your Binary search and just state what is the number of comparisons required in such a case? Do an average case analysis for Binary Search on the above array.

5 + (2+5) = 12

Group – D

6.(a) Consider a DFS on a directed graph. What are tree edges, back edges, forward edges and cross edges?

(b) Give the pseudo code for doing DFS on a directed graph. Can you add a few lines of pseudo code to this routine so that it can print the type of each edge while doing the traversal?

(d) If you are doing a DFS on an undirected graph, you may not be able to identify all the four types of edges; however, there will still be two types of edges. Which are they? Explain why the other two types of edges will vanish.

4 + 6 + 2 = 12

7.(a) Give the pseudo-code for Euclid's GCD algorithm and illustrate it with one small example.

(b) What problem does Floyd-Warshall algorithm solve and what is its time Complexity? What is the problem if we try to use Bellman-Ford algorithm to solve such a problem?

(c) Prove that subpaths of shortest paths are also shortest paths.

(4 + 2) + 3 + 3 = 12

Group – E

8.(a) Explain KMP algorithm for pattern matching with the help of the following String S and Pattern P

S: **babcbabcabcaabcabcabcacabc**

P: **abcabcacab**

(b) Differentiate between NP-Complete and NP-Hard problems. Show that CLIQUE problem is NP-Complete.

6+(2+4)= 12

9.(a) A flow in G is a real valued function f: V x V $\rightarrow$ R that satisfies 3 properties. What are they and also state each of them in one sentence.

(b) Give a polynomial-time 2-approximation algorithm for vertex cover problem and also prove that the algorithm indeed achieves that factor.

3 + (5 + 4) = 12