M.TECH/CSE/1ST SEM /MATH 5102/2015
2015

Advanced Discrete Mathematics
(MATH 5102)

Time Allotted : 3 hrs 19 **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternatives for the following: **10 x 1=10**

(i)Number of functions from a set with m elements to a set with n elements is

(a) n^m (b) m^n (c) nm (d) $n!m!$.

(ii) The equation $12x + 15y = 28$

(a) has a unique solution in integers (b) has two solutions in integers
(c) has no solution in integers (d) has infinite number of solutions in integers.

(iii) The number of maximal elements in the poset $\{1,2,3,6,12,24,36,48\}$ are (with respect to divisibility relation)

(a) 3 (b) 1 (c) 2 (d) 4.

(iv) Among the following posets, the set that is totally ordered is

(a) $(C, \leq)$ (b) $(\{1,2,4,6\},|)$
(c) $(\{2,4,6,8,10\},|)$ (d) $(Z, \leq)$.

(v) The relation $\{ (1,2), (1,3), (3,1), (1,1), (3,3), (3,2), (1,4), (4,2), (3,4)\}$ is

(a) reflexive (b) transitive (c) symmetric (d) anti-symmetric.

(vi) The remainder when the sum 4! +5! + 6! +.....+50! is divided by 4 is

(a) 1 (b) 2 (c) 3 (d) 0.

(vii) A graph consists of three components, each of which is a tree having 6 vertices.The chromatic number of the graph is

(a) 3 (b) 2 (c) 6 (d)18 .

(viii) If $\chi(G) = 15$, then the maximum number of colours required for colouring G is

(a) 13 (b) 14 (c) 15 (d) None of these.

(ix) The chromatic polynomial of a tree having 10 vertices is

(a) $x(x+1)^{10}$ (b) $(x-1)^9$
(c) $x^9(x-1)$ (d) $x(x-1)^9$.

(x) Which of the following function $f: Z \times Z \to Z$ is not surjective

(a) $f(a,b) = a+b$ (b) $f(a,b) = a-b$

(c) $f(a,b) = a$ (d) $f(a,b) = |b|$.

Group – B

2.(a) Draw the Hasse diagram for the set of positive integer divisors of 32 with respect to the divisibility relation.

(b) Show that there is exactly one greatest element of a poset, if such an element exists.

(c) What is the covering relation of the partial ordering $\{(a,b) \mid a \text{ divides } b\}$ on $\{1,2,3,4,6,12\}$.

4 + 4 + 4 = 12

3.(a) Prove that the set $\{\phi, \{a\}, \{a,c\}, \{c\}, \{a,b,c\}\}$ is a lattice with respect to the relations $\cap$ and $\cup$. Is it complemented?

(b) Prove that a poset is a lattice if the supremum & infimum of (a,b) exist for every pair of elements a, b in the set.

6 + 6 = 12

Group – C

4.(a) Find gcd(12378,3054) and express it as $12378x + 3054y$, where x and y are integers.

(b) Show that $4^{203} \equiv 4 \ (mod\ 5)$. State any theorem that you use.

6 + 6 =12

5.(a) If $\gcd(a,b) = 1$, prove that $\gcd(a^2 - b^2, a^2 + b^2) = 1$ or 2.

(b) Prove that, if c is a divisor of ab and $\gcd(b,c) = 1$, then c is a divisor of a.

6 + 6 =12

Group – D

6.(a) Find a general solution of the recurrence relation

$$a_n = 4a_{n-1} - 3a_{n-2} \qquad (n \geq 2)$$

(b) Find the coefficient of X^{23} and X^{32} in $(1 + X^5 + X^9)^{10}$

(d) If A and B are some subsets of some universal set U, then prove that

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

5 + 4 + 3 = 12

7.(a) How many different license plates are available if each plate contains a sequence of three letters followed by three digits?

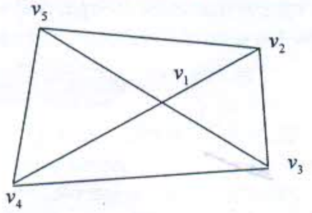
(b) Show that among any $(n+1)$ positive integers not exceeding $2n$ there must be an integer that divides one of the other integers.

(c) Prove that ${}^{n+g-1}C_n = {}^gC_1 \times {}^{n-1}C_0 + {}^gC_2 \times {}^{n-1}C_1 + \cdots\cdots + {}^gC_g \times {}^{n-1}C_{g-1}$ where $n > g$.

4 + 5 + 3 = 12

Group – E

8.(a) Find the chromatic polynomial of the following graph:



(b) Prove that a bipartite graph cannot contain a cycle of odd length.

6 + 6 =12

9.(a) Prove that a graph G is a tree if and only if every pair of vertices in it is connected by one and only one path.

(b) If every region of a simple planar graph having n vertices and e edges drawn in a plane is bounded by k edges, prove that $k(n-2) = e(k-2)$.

6 + 6 =12