

MATHEMATICAL & STATISTICAL METHODS (MTH2204)

Time Allotted : 2½ hrs

Full Marks : 60

Figures out of the right margin indicate full marks.

***Candidates are required to answer Group A and
any 4 (four) from Group B to E, taking one from each group.***

Candidates are required to give answer in their own words as far as practicable.

Group – A

1. Answer any twelve:

12 × 1 = 12

Choose the correct alternative for the following

- (i) Lagrange's method is based on solving the characteristic equations of the form:
 (a) $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$ (b) $\frac{dx}{P} + \frac{dy}{Q} = \frac{dz}{R}$
 (c) $Pdx + Qdy + Rdz = 0$ (d) $Pdx + Qdy = Rdz$
- (ii) For the differential equation $(x^2 - 9)^2 y'' + (x - 3)y' + 2y = 0$ a regular singular point is
 (a) $x = 3$ (b) $x = -3$ (c) $x = 0$ (d) $x = 1$
- (iii) Newton's Divided Difference Interpolation is particularly useful for:
 (a) Unequally spaced data points.
 (b) Equally spaced data points only.
 (c) Extrapolation beyond given data points.
 (d) Data sets with missing values.
- (iv) If Δ is the forward difference operator, then the second-order difference is given by:
 (a) $\Delta^2 f(x) = f(2x + h) - f(x)$ (b) $\Delta^2 f(x) = f(x^2 + h) - f(x)$
 (c) $\Delta^2 f(x) = f(x + 2h) - 2f(x + h) + f(x)$ (d) $\Delta^2 f(x) = f(x + h) + 2f(x)$
- (v) Which of the following numerical integration rules requires the number of intervals n to be a multiple of 6?
 (a) Trapezoidal rule (b) Simpson's 1/3 rule
 (c) Simpson's 3/8 rule (d) Weddle's rule
- (vi) The value of a_0 in the Fourier series of $f(x) = x^3 \cos 2x$ in $(-2\pi, 2\pi)$ is
 (a) 0 (b) 1 (c) 2π (d) $8\pi^3$
- (vii) If a function $f(x)$ is even, which terms will be present in its Fourier series expansion?
 (a) Only sine terms. (b) Only cosine terms.
 (c) Both sine and cosine terms. (d) None of the above.

- (viii) A function $f(x)$ is defined in the interval $(-\pi, \pi)$ such that $f(x) = x^2 \sin x$. The Fourier coefficient a_n for the function will be:
 (a) $2/\pi$. (b) Always zero.
 (c) Non-zero for all n . (d) Equal to b_n .
- (ix) In a Hypergeometric Distribution, the probability of success is determined without:
 (a) Replacement. (b) Sampling.
 (c) Random selection. (d) Independence.
- (x) In a bivariate data set, if the two regression lines are given by $y = 0.5x + 10$ and $x = 0.8y + 2$, what is the value of the Pearson correlation coefficient (r)?
 (a) 0.4 (b) 0.632 (c) 0.360 (d) 1.3

Fill in the blanks with the correct word

- (xi) Charpit's method is used to solve first-order non-linear partial differential equations of the form $F(x, y, z, p, q) = 0$, where p and q represent the partial derivatives of z with respect to _____ and _____.
- (xii) The Trapezoidal Rule for numerical integration approximates the integral of a function by dividing the area under the curve into _____ trapezoids.
- (xiii) The term orthogonality in Fourier Series refers to the property that *sine* and *cosine* functions are mutually _____ over a given interval.
- (xiv) Skewness measures the _____ of a distribution, indicating whether it is symmetric or not.
- (xv) In a frequency distribution, if the coefficient of skewness γ_1 is found to be zero, the distribution is said to be _____.

Group - B

2. (a) Find a complete integral of $q = (z + px)^2$ where $p = \frac{\partial z}{\partial x}$, $q = \frac{\partial z}{\partial y}$.
 [(MTH2204.5)(Analyse/IOCQ)]
- (b) Solve $\frac{y^2 z}{x} p + xzq = y^2$ where $p = \frac{\partial z}{\partial x}$, $q = \frac{\partial z}{\partial y}$.
 [(MTH2204.5)(Evaluate/HOCQ)]
- 6 + 6 = 12**
3. (a) Obtain a partial differential equation by eliminating the arbitrary constants from the relation $z = a(x + y) + b(x - y) + abz + c$ where x and y are independent variables, and a, b, c are arbitrary constants.
 [(MTH2204.5)(Understand/LOCQ)]
- (b) Find a partial differential equation by elimination of arbitrary function ϕ from $z = \phi\left(\frac{y}{x}\right)$.
 [(MTH2204.5)(Remember/LOCQ)]
- (c) Solve $x^2 p + y^2 q = z^2$ where $p = \frac{\partial z}{\partial x}$, $q = \frac{\partial z}{\partial y}$.
 [(MTH2204.5)(Analyse/IOCQ)]
- 4 + 4 + 4 = 12**

Group - C

4. (a) Using Lagrange's interpolation formula calculate the value of y when $x = 103$ from the following table

$x:$ 93.0 96.2 100.0 104.2 108.7

$y:$ 11.38 12.80 14.70 17.07 19.91.

[[MTH2204.1, MTH2204.2](Analyse/IOCQ)]

- (b) A function $f(x)$ is given as a table of values. Approximate the area under the curve $y = f(x)$ from $x = 1$ to $x = 3$ using Simpson's $\frac{1}{3}$ rd rule.

x	1	1.5	2	2.5	3
$f(x)$	5.88	8.76	12.47	18.2	25.5

[[MTH2204.1, MTH2204.2](Remember/LOCQ)]

6 + 6 = 12

5. (a) Using Newton's divided difference formula, calculate the value of y when $x = 2.5$ for the following table:

x :	0	1	2	4	5	6
y :	1	14	15	5	6	19

[[MTH2204.1, MTH2204.2](Evaluate/HOCQ)]

- (b) Apply Weddle's rule to evaluate $\int_0^6 \frac{dx}{(1+x)^2}$, taking six equal sub-intervals of $[0,6]$, correct to three decimal places.

[[MTH2204.1, MTH2204.2](Apply/IOCQ)]

6 + 6 = 12

Group - D

6. (a) Find the Fourier series of the function $f(x) = \begin{cases} 1, & 0 < x < 1 \\ 2, & 1 < x < 2 \end{cases}$. Hence obtain the value of $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$.

[[MTH2204.4](Analyse/IOCQ)]

- (b) Using Parseval's Identity for half-range *sine* series of the function $f(x) = x(\pi - x)$ in $0 < x < \pi$, prove that $\frac{1}{1^6} + \frac{1}{3^6} + \frac{1}{5^6} + \dots = \frac{\pi^6}{960}$.

[[MTH2204.4](Evaluate/HOCQ)]

6 + 6 = 12

7. (a) Determine the radius of convergence and the exact interval of convergence for the following power series:

$$\sum_{n=1}^{\infty} \frac{(-1)^n (x-2)^n}{n \cdot 3^n}$$

[[MTH2204.4](Analyse/IOCQ)]

- (b) A half-wave rectifier converts an alternating current into a pulsating direct current by clipping the negative half of the cycle. Consider the periodic function $f(t)$ with period $T = 2\pi$ defined as:

$$f(t) = \begin{cases} A \sin t, & 0 < t < \pi \\ 0, & \pi < t < 2\pi \end{cases}$$

- (i) Calculate the **Euler coefficients** a_0 , a_n , and b_n for this function.

(ii) Write the first three non-zero terms of the Fourier series expansion.

[[MTH2204.4](Evaluate/HOCQ)]

6 + 6 = 12

Group - E

8. (a) The following table gives the distribution of 100 families according to daily expenditure:

Expenditure (in Rs.)	0-10	11-20	20-30	30-40	40-50
Number of families	14	x	27	y	15

If the mode of the distribution is 24, find the missing frequencies x and y .

[[MTH2204.3, MTH2204.6](Evaluate/HOCQ)]

- (b) Calculate the correlation co-efficient between x and y for the following data:

x	1	2	3	4	5	6	7	8	9
y	9	8	7	6	5	4	3	2	1

[[MTH2204.3, MTH2204.6](Apply/IOCQ)]

6 + 6 = 12

9. (a) Average number of accidents on any day on a national highway is 1.8. Determine the probability that the number of accidents are: (i) at least one (ii) at most one. (Given: $e^{-1.8} = 0.16529$)

[[MTH2204.3, MTH2204.6](Evaluate/HOCQ)]

- (b) Calculate the mean and standard deviation of the following frequency distribution:

Class	10 – 19	20 – 29	30 – 39	40 – 49	50 – 59	60 – 69
Frequency	3	5	7	9	4	2

[[MTH2204.3, MTH2204.6](Analyse/IOCQ)]

6 + 6 = 12

Cognition Level	LOCQ	IOCQ	HOCQ
Percentage distribution	14.6	47.9	37.5