M.TECH/AEIE/1ST SEM /MATH 5101/2015
2015

Advanced Mathematical Methods
(MATH 5101)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternatives for the following: **10 x 1=10**

(i) The product of the eigen values of a square matrix A is
(a) 0 (b) 1 (c) detA (d) none of these.

(ii) A basis of a vector space is
(a) a minimal spanning subset
(b) a maximal spanning subset
(c) a linearly independent subset
(d) a linearly dependent subset.

(iii) Let V and W be finite dimensional vector spaces over a field F and $T: V \rightarrow W$ is a linear mapping. If dimV=3 and dimension of KerT=2, then Rank of T is
(a) 1 (b) 2 (c) 3 (d) 0.

(iv) If $\lambda_i, i=1,2,3$ are the eigen values of the matrix $\begin{bmatrix} 1 & 1 & 2 \\ 0 & 3 & 2 \\ 1 & 3 & 9 \end{bmatrix}$

(a) $\sum_{i=1}^{3} \lambda_i = 13$ (b) $\sum_{i=1}^{3} \lambda_i = 27$
(c) $\sum_{i=1}^{3} \lambda_i = 17$ (d) $\sum_{i=1}^{3} \lambda_i = 22$

(v) Let V and W be finite dimensional vector spaces over a field F and $T: V \rightarrow W$ is a linear mapping. Then
(a) KerT is a subspace of W (b) KerT is a subspace of V
(c) ImT is a subspace of V (d) none of these.

(vi) In general, Lagrange's Multiplier method is
(a) Necessary and Sufficient
(b) Sufficient
(c) Necessary
(d) None of the above.

(vii) If the feasible set of an optimization problem is unbounded, then
(a) no finite optimum point exists.
(b) it has an infinite number of feasible points.
(c) the existence of a finite optimum point cannot be assured.
(d) None of these.

(viii) The set S={(x,y): $x^2+y^2=25$} is
(a) Concave (b) Convex (c) both (a) and (b) (d) none of these.

(ix) The critical points of the function $f(x,y)=x^3+3xy^2-15x^2-15y^2+72x$ are
(a) (6,0), (8,0), (1,5), (5,1) (b) (6,0), (4,0), (5,1), (5,-1)
(c) (6,0), (4,0), (1,5), (1,-5) (d) (6,0), (8,0), (-1,-5), (1,5).

(x) The dual of a dual is
(a) dual (b) primal (c) neither primal nor dual (d) none of these.

Group – B

2.(a) Let V be the vector space of function $f: R \to R$. Show that W=$\{f(x): f(-x)=-f(x), \forall x \in R\}$ is a subspace of V.

(b) Determine whether or not the vectors u=(1,1,2), v=(2,3,1) and w=(4,5,5) in R^3 are linearly independent.

6 + 6 = 12

3.(a) Use Gram-Schmidt process to obtain an orthogonal basis from the basis set $\{(1,0,1),(1,1,1),(1,3,4)\}$ of the Euclidean space $\Re^3$ with standard inner product.

(b) Prove that, the set of vectors $\{(2,1,1),(1,2,2),(1,1,1)\}$ is linearly dependent in $\Re^3$.

6 + 6 = 12

Group – C

4.(a) Determine the linear mapping $T:\Re^3 \to \Re^3$ which maps the basis vectors (0,1,1),(1,0,1),(1,1,0) of $\Re^3$ to (1,1,1),(1,1,1),(1,1,1) respectively.

(b) Verify the rank-nullity theorem for T.

6 + 6 = 12

5.(a) Let $T: R^2 \to R^2$ be a linear transformation such that $T(1,1)=(1,3)$ and $T(-1,1)=(3,1)$. Find $T(a,b)$, for any $(a,b) \in R^2$.

(b) Is a subset of a linearly dependent set always linearly dependent? Justify your answer, if required with suitable example.

6 + 6 = 12

Group – D

6.(a) Solve the following non-linear programming problem using Lagrange multiplier method:

Maximize $f(x_1, x_2, x_3) = 2x_1^2+x_2^2+3x_3^2+10x_1+8x_2+6x_3-100$

Subject to $x_1+x_2+x_3=20$

$x_1, x_2, x_3 \ge 0$

(b) Verify whether the function $f(x_1, x_2) = 2x_1^3 - 6x_2^2$ is concave or convex.

8 + 4 =12

7. Consider the following problem:

Minimize $f = (x_1 - 1)^2 - x_2^2$

Subject to $x_1^3 - 2x_2 \le 0,$
$x_1^3 + 2x_2 \le 0,$

Determine whether the constraint qualification and the Kuhn-Tucker conditions are satisfied at the optimal point.

12

Group – E

8.(a) Solve the following LPP using simplex method:

Minimize $Z = x_1-3x_2+2x_3$

Subject to $3x_1-x_2+2x_3 \le 7$

$-2x_1+4x_2 \le 12$

$-4x_1+3x_2+8x_3 \le 10$

$x_1, x_2, x_3 \ge 0$

(b) Find the dual of the following problem:

Maximize $Z = 2x_1+3x_2-4x_3$

Subject to $3x_1+x_2+x_3 \le 2$

$-4x_1+3x_3 \ge 4$

$x_1-5x_2+x_3 = 5$

$x_1, x_2 \ge 0$ and x_3 is unrestricted

8 + 4 = 12

9.(a) Use simplex method to solve the following LPP:

Maximize $z = 3x_1 - x_2,$ Subject to $-x_1 + 3x_2 \ge 2$
$5x_1 - 2x_2 \ge 2$
$x_1, x_2 \ge 0$, by Charnes Big M method.

(b) Construct the dual of the following LPP:

Minimize $z = x_1 - x_2,$ Subject to $2x_1 + x_2 \ge 12,$
$-x_1 - x_2 \ge 1,$
$x_1, x_2 \ge 0.$

8 + 4 = 12