

**ADVANCED DISCRETE MATHEMATICS AND STATISTICAL METHODS  
(MTH5101)**

Time Allotted : 2½ hrs

Full Marks : 60

*Figures out of the right margin indicate full marks.*

*Candidates are required to answer Group A and  
any 4 (four) from Group B to E, taking one from each group.*

*Candidates are required to give answer in their own words as far as practicable.*

**Group – A**

1. Answer any twelve:

12 × 1 = 12

*Choose the correct alternative for the following*

- (i) The probability that a leap year will have 53 Fridays or 53 Saturdays is  
 (a)  $\frac{1}{7}$                       (b)  $\frac{2}{7}$                       (c)  $\frac{3}{7}$                       (d)  $\frac{4}{7}$
- (ii) Three numbers are chosen at random from 1 to 20. The probability that they are consecutive is  
 (a)  $\frac{5}{190}$                       (b)  $\frac{1}{190}$                       (c)  $\frac{1}{120}$                       (d)  $\frac{3}{190}$
- (iii) The random variable  $X$  has distribution function  $F$ . If  $F(0) = \frac{1}{6}$ ,  $F(1) = \frac{1}{2}$ ,  $F(3) = \frac{3}{4}$ , then  $P(0 < X \leq 3)$  is  
 (a)  $\frac{3}{4}$                       (b)  $\frac{1}{3}$                       (c)  $\frac{7}{12}$                       (d)  $\frac{2}{3}$
- (iv) If  $F(x)$  is the distribution function of a random variable, then  
 (a)  $F(x)$  is continuous at all points    (b)  $F(x)$  is monotonic decreasing function  
 (c)  $F(-\infty) = 1$                       (d)  $F(+\infty) = 1$ .
- (v) Which of the following can be taken as the probability distribution of a discrete random variable for the range 1, 2, 3, 4?  
 (a)  $p(x) = \frac{x^2}{15}$                       (b)  $p(x) = \frac{x-2}{2}$   
 (c)  $p(x) = \frac{x+2}{18}$                       (d)  $p(x) = \frac{x-3}{2}$
- (vi) The generating function of the infinite sequence  $\{1, 0, 1, 0, 1, 0, 1, 0, \dots\}$   
 (a)  $\frac{1}{1+x}$                       (b)  $\frac{1}{1-x^2}$                       (c)  $\frac{1}{1+x^2}$                       (d)  $\frac{1}{1-x-x^2}$
- (vii) The solution of the recurrence relation  $a_n = 2a_{n-1} + 1$  with  $a_0 = 0$  is  
 (a)  $1 - 2^n$                       (b)  $2^n - 2$                       (c)  $2^{n-1} - 1$                       (d)  $2^n - 1$
- (viii) The number of non-empty subsets of a set having 8 elements is  
 (a) 255                      (b) 256                      (c) 254                      (d) 253

- (ix) Let  $K_n$  denote the complete graph having  $n$  vertices. Which one of the following is an Eulerian graph?  
 (a)  $K_4$                       (b)  $K_5$                       (c)  $K_6$                       (d)  $K_8$
- (x) How many colours are necessary for colouring a graph having 8 vertices and no edge?  
 (a) 8                              (b) 7                              (c) 3                              (d) 1

*Fill in the blanks with the correct word*

- (xi) The probability that the sum 8 appears in a single toss of a pair of a fair dice is \_\_\_\_\_.
- (xii)  $E(XY) = E(X)E(Y)$  if the two random variables  $X$  and  $Y$  are independent.
- (xiii) The tenth Fibonacci number is \_\_\_\_\_.
- (xiv) The chromatic number of a tree having 6 vertices is \_\_\_\_\_.
- (xv) The generating function of the infinite sequence  $\{1, -1, 1, -1, 1, -1, 1, -1, \dots\}$  is \_\_\_\_\_.

### Group - B

2. (a) In a shooting competition the probability of a man hitting a target is  $\frac{1}{5}$ . If he fires 5 times, what is the probability of hitting the target at least twice?  
[[MATH 5101.1, MATH5101.2](Understand/LOCQ)]
- (b) A die is tossed thrice. A success is 'getting 1 or 6' on a toss. Find the mean and variance of the number of successes.  
[[MATH 5101.1, MATH5101.2](Evaluate/HOCQ)]  
**6 + 6 = 12**
3. (a) In an examination 30% of the student failed in mathematics, 15% in chemistry and 10% in both chemistry and mathematics. A student is selected at random if he failed in chemistry. What is the probability that he passed in mathematics?  
[[MATH5101.1, MATH5101.2](Understand/LOCQ)]
- (b) A box contains 5 defective and 10 non-defective lamps. 8 lamps are drawn at random without replacement. What is the probability that the 8th lamp is the 5<sup>th</sup> defective?  
[[MATH5101.1, MATH5101.2](Analyse/IOCQ)]  
**6 + 6 = 12**

### Group - C

4. (a) A discrete random variable  $X$  has the mean 6 and variance 2. Assuming the distribution is binomial, find the probability that  $5 \leq X \leq 7$ .  
[[MATH5101.1, MATH5101.2](Apply/IOCQ)]
- (b) Assuming that the height of group is normally, find the mean and standard deviation if 84% of the mean have height less than 65.2 inches and 68% have height lying between 65.2 and 62.8 inches. [Given  $\Phi(0.9) = 0.854$ ,  $\Phi(-0.9) = 0.16$ ]  
[[MATH5101.1, MATH5101.2](Remember/LOCQ)]  
**6 + 6 = 12**

5. (a)  $A$  and  $B$  play a game in which their chances of winning are in the ratio 3:2. Find  $A$ 's chances of winning at least three games out of the five games played.  
 [(MATH5101.1,MATH5101.2)(Analyse/IOCQ)]
- (b) The arithmetic mean and standard deviation of a series of 20 items were calculated by a student as 25 and 6 respectively. But while calculating an item, 14 was misread as 40. Find the correct arithmetic mean and standard deviation.  
 [(MATH5101.1,MATH5101.2)(Understand/LOCQ)]

**6 + 6 = 12**

### Group - D

6. (a) If there are 200 faculty members that speak French, 50 that speak Russian, 100 that speak Spanish, 20 that speak French and Russian, 60 that speak French and Spanish, 35 that speak Russian and Spanish, while only 10 speak French, Russian and Spanish, how many speak either French or Russian or Spanish?  
 [(MATH5101.1,MATH5101.2,MATH5101.3)(Analyse/IOCQ)]
- (b) Solve the following recurrence relation by the method of generating functions:  
 $a_n - 7a_{n-1} + 10a_{n-2} = 0$ , for  $n \geq 2$ , where  $a_0 = 10, a_1 = 41$ .  
 [(MATH5101.1,MATH5101.2,MATH5101.3)(Apply/IOCQ)]
7. (a) How many bit strings of length 10 contain (i) exactly four 1's; (ii) at most four 1's; (iii) at least four 1's.  
 [(MATH5101.1,MATH5101.2,MATH5101.3)(Understand/LOCQ)]
- (b) Prove combinatorially that the number of subsets of a set having  $n$  elements is  $2^n$ .  
 [(MATH5101.1,MATH5101.2,MATH5101.3)(Analyse/IOCQ)]

**6 + 6 = 12**

**6 + 6 = 12**

### Group - E

8. (a) Using the Decomposition theorem, find the chromatic polynomial and hence the chromatic number of the graph:

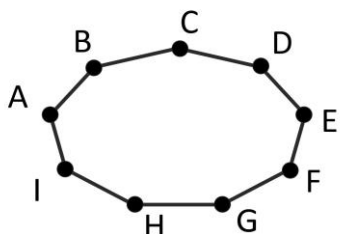


[(MATH5101.1,MATH5101.2,MATH5101.4)(Apply/IOCQ)]

- (b) Draw any six spanning trees of  $K_4$ , the complete graph having four vertices.  
 [(MATH5101.1,MATH5101.2,MATH5101.4)(Understand/LOCQ)]

**6 + 6 = 12**

9. (a) Identify all distinct maximum matchings of size five in the following graph,  $C_9$  contain the edge  $ab$ . Justify your answer.



[(MATH5101.1,MATH5101.2,MATH5101.4)(Analyse/IOCQ)]

(b) Explain why neither of the following polynomials can be a chromatic polynomial,

(i)  $\lambda^3 + 5\lambda^2 - 3\lambda + 5$       (ii)  $\lambda^4 + 3\lambda^3 - 3\lambda^2$

*[(MATH5101.1, MATH5101.2, MATH5101.4)(Analyse/IOCQ)]*

**6 + 6 = 12**

---

Cognition Level	LOCQ	IOCQ	HOCQ
Percentage distribution	37.5	56.25	6.25