INTRODUCTION TO OPTIMIZATION
(MTH2104)

Time Allotted : 2½ hrs **Full Marks : 60**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 4 (four) from Group B to E, taking one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A

1. Answer any twelve: **12 × 1 = 12**

Choose the correct alternative for the following

(i) A feasible solution to an LP problem
(a) must satisfy all the problem's constraints
(b) need not satisfy all the constraints, only some of them
(c) must be a corner point of the feasible region
(d) must optimize the value of the objective function.

(ii) If an optimal solution is degenerate, then
(a) there are alternative optimal solutions
(b) the solution is infeasible
(c) the solution is of no use to the decision-maker
(d) the formulation of the LPP is incorrect.

(iii) Each constraint in an LP model in standard form is expressed as an
(a) inequality with $\geq$ sign
(b) inequality with $\leq$ sign
(c) equality with $=$ sign
(d) inequality with $<$ sign.

(iv) Which of the following is not the characteristic of linear programming?
(a) Resources must be limited
(b) Only one objective function
(c) Parameters value remains constant during planning period
(d) The problem must be of maximization type.

(v) In an assignment problem, k is the maximum number of zeros which can be assigned, then the minimum number of lines which will cover all the zeros is
(a) $k-1$ (b) k (c) $k+1$ (d) $2k$.

(vi) If there were n workers and n jobs in an assignment problem, there would be
(a) $(n-1)!$ Solutions
(b) $n!$ solutions
(c) $(n+1)!$ Solutions
(d) n solutions.

(vii) In the Hungarian method, if the number of rows is not equal to the number of columns:
(a) the problem has no solution
(b) we balance it by adding dummy rows or columns with zero cost
(c) the method cannot be applied
(d) only North-West Corner rule can be used.

(viii) For a two-person game of A and B, the pay-off matrix is given for B. The optimum strategies are
(a) minimax for A, maximin for B
(b) maximin for A, minimax for B
(c) minimin for A, maximin for B
(d) maximin for A, minimin for B.

(ix) A saddle point exists when
(a) maximin value = maximax value
(b) minimax value = maximax value
(c) minimax value = maximin value
(d) minimax value ≠ maximin value.

(x) Activity in a network diagram is represented by
(a) arrows (b) circles (c) squares (d) rectangles.

Fill in the blanks with the correct word

(xi) In the optimal simplex table, $C_j - Z_j = 0$ value indicates _____ solution.

(xii) When there is no finite solution in the LP problem, the solution is _____.

(xiii) The graphical method of solving an LPP can be applied when the number of decision variables is _____.

(xiv) If S is a closed convex set and x is extreme point of S, then $S - \{x\}$ is _____.

(xv) Johnson's Rule is applicable for _____ machines, many jobs.

Group - B

2. (a) A retail store stocks two types of shirts A and B. These are packed in attractive cardboard boxes. During a week the store can sell a maximum of 400 shirts of type A and a maximum of 300 shirts of type B. The storage capacity, however, is limited to a maximum of 600 of both types combined. Type A shirt fetches a profit of Rs. 2/- per unit and type B a profit of Rs. 5/- per unit. How many of each type the store should stock per week to maximize the total profit? Formulate a mathematical model of the problem. *[(MTH2104.1, MTH2104.4)(Understand/LOCQ)]*

(b) Solve by simplex method.

$$\text{Maximize } Z = 2x_1 - 3x_2 + 5x_3,$$

subject to the constraints

$$x_1 + x_2 + x_3 \leq 20,$$
$$2x_1 + x_2 \leq 16,$$
$$x_2 + x_3 \leq 10,$$
$$x_1, x_2, x_3 \geq 0.$$

[(MTH2104.1, MTH2104.4)(Evaluate/HOCQ)]

4 + 8 = 12

3. (a) Show that $S = \{(x_1, x_2): 4x_1 + 5x_2 = 13\} \in \mathrm{R}^2$ is a convex set. *[(MTH2104.1, MTH2104.4)(Understand/LOCQ)]*

(b) Use the Simplex method to find the maximum value of

$$Z = 2x_1 - x_2 + 2x_3,$$

subject to the constraints

$$2x_1 + x_2 \leq 10,$$
$$x_1 + 2x_2 - 2x_3 \leq 20,$$
$$x_2 + 2x_3 \leq 5,$$
$$\text{where } x_1, x_2, x_3 \geq 0.$$

[(MTH2104.1, MTH2104.4)(Evaluate/HOCQ)]

4 + 8 = 12

Group - C

4. (a) A company has four manufacturing plants P_1, P_2, P_3, P_4 and five warehouses W_1, W_2, W_3, W_4, W_5. Each plant manufactures the same product, which is sold at different prices in each warehouse area. The capacities of the plants are 100, 200, 120 and 80 respectively. Demands of warehouses are 80, 120, 150, 70 and 90 respectively. The profit (Rs.) per unit product taken from different plant to different warehouses are given bellow:

Plants \ Warehouses	W_1	W_2	W_3	W_4	W_5
P_1	6	4	6	4	8
P_2	6	6	4	10	8
P_3	11	10	7	14	7
P_4	15	12	6	14	9

(i) Formulate this problem as a transportation problem in order to maximize profit.
(ii) Find the initial solution using Vogel's approximation method (VAM).
(iii) Test for optimality and find the optimal solution. *[(MTH2104.1, MTH2104.2)(Analyse/IOCQ)]*

(b) Give the economic interpretation of u_i's and v_j's in Transportation Problem. *[(MTH2104.1, MTH2104.2)(Remember/LOCQ)]*

(3 + 4 + 3) + 2 = 12

5. (a) A department of a company has five employees with five jobs to be performed. The time (in hours) that each man takes to perform each job is given in the effectiveness matrix.

Jobs \ Employees	I	II	III	IV	V
A	10	5	13	15	16
B	3	9	18	13	6
C	10	7	2	2	2
D	7	11	9	7	12
E	7	9	10	4	12

How should the jobs be allocated, one per employee, so as to minimize the total man-hours? *[(MTH2104.1, MTH2104.2)(Apply/IOCQ)]*

(b) Solve the following integer programming problem using branch and bound method.

$$\text{Maximize } Z = 5x_1 + 7x_2,$$

subject to the constrained

$$2x_1 + x_2 \leq 13,$$
$$5x_1 + 9x_2 \leq 41,$$
$$x_1, x_2 \geq 0 \text{ are integers.}$$

[(MTH2104.1, MTH2104.2)(Evaluate/HOCQ)]

6 + 6 = 12

Group - D

6. (a) Use the graphical method to obtain the optimal strategies for both persons and the value of the game for two-person zero-sum game, whose payoff matrix is as follows:

Player A \ Player B	B1	B2
A1	1	−3
A2	3	5
A3	−1	6
A4	4	1
A5	2	2
A6	−5	0

[(MTH2104.1, MTH2104.3)(Apply/IOCQ)]

(b) Explain the principal of dominance in game theory and solve the following game

Player A \ Player B	B1	B2	B3	B4	B5
A1	2	4	3	8	4
A2	5	6	3	7	8
A3	6	7	9	8	7
A4	4	2	8	4	3

[(MTH2104.1, MTH2104.3)(Evaluate/HOCQ)]

6 + 6 = 12

7. (a) Compare the role of centralized control in traditional and distributed databases. *[(MTH2104.1, MTH2104.3)(Analyse/IOCQ)]*

(b) Use algebraic method to solve the following game.

Player A \ Player B	B_1	B_2	B_3
A_1	−1	2	1
A_2	1	−2	2
A_3	3	4	−3

[(MTH2104.1, MTH2104.3)(Apply/IOCQ)]

6 + 6 = 12

Group - E

8. (a) What is the significance of critical path in project diagram? *[(MTH2104.1, MTH2104.5, MTH2104.6)(Remember/LOCQ)]*

(b) An established company has decided to add a new product to its line. It will buy the product from a manufacturing concern, package it, and sell it to a number of distributors that have been selected on a geographical basis. Market research has already indicated the volume expected and the size of sales force required. The steps shown in the following table are to be planned.

Activity	Description	Predecessor Activity	Duration (Days)
A	Organize sales office	—	6
B	Hire salesmen	A	4
C	Train salesmen	B	7
D	School advertising agency	A	2
E	Plan advertising campaign	D	4
F	Conduct advertising campaign	E	10
G	Design package	—	2
H	Setup packaging facilities	G	10
I	Package initial stocks	J, H	6
J	Order stock from manufacturer	—	13
K	Select distributors	A	9
L	Sell to distributors	C, K	3
M	Ship slacks 10 distributors	I, L	5

(i) Draw a network diagram for this project.

(ii) Indicate the critical path. *[(MTH2104.1, MTH2104.5, MTH2104.6)(Understand/LOCQ)]*

3 + 9 = 12

9. (a) The sales manager of Domestic Products Limited was informed by the company's R&D department about the completion of the prototype of a particular product. He consulted the production manager on the time taken to produce the first batch of the product, which is needed for demonstration in his sales promotion programme. He also decided to invite a few industrial representatives to the demonstration of this new product and through them to launch it in the market. The various activities involved in this marketing project, their descriptions, estimated duration (in days) and immediate predecessors are given in the following table:

Activity	Description	Time (days)	Immediate Predecessors
A	Collect data on specifications and capabilities	4	–
B	Prepare operation manual	4	A
C	Chart out promotion program	4	B
D	Make copies of manual and promotion material	9	B
E	Produce first batch for demonstration	16	B
F	Prepare list of press representatives	2	C
G	Chief executive's conference with Managers	1	C
H	Press representatives reach Bombay	2	F, G
I	Promotional meetings	4	D, H
J	Product demonstration	2	E, I
K	Press representatives return home	2	J

(i) Draw the network diagram for the project.
(ii) Identify the critical path. *[(MTH2104.1, MTH2104.5, MTH2104.6)(Understand/LOCQ)]*

(b) There are seven jobs, each of which has to go through the machines A and B in the order AB. Processing times in hours are as follows:

Job:	1	2	3	4	5	6	7
Machine A:	3	12	15	6	10	11	9
Machine B:	8	10	10	6	12	1	3

Determine a sequence of these jobs that will minimize the total elapsed time. *[(MTH2104.1, MTH2104.5, MTH2104.6)(Analyse/IOCQ)]*

7 + 5 = 12

Cognition Level	LOCQ	IOCQ	HOCQ
Percentage distribution	29.17	41.67	29.16