

MATHEMATICAL FOUNDATIONS
(MCA1105)

Time Allotted : 2½ hrs

Full Marks : 60

Figures out of the right margin indicate full marks.

*Candidates are required to answer Group A and
any 4 (four) from Group B to E, taking one from each group.*

Candidates are required to give answer in their own words as far as practicable.

Group – A

1. Answer any twelve:

12 × 1 = 12

Choose the correct alternative for the following

- (i) The relationship between the Beta and Gamma functions is given by
 (a) $\beta(m, n) = \frac{\Gamma(m)+\Gamma(n)}{\Gamma(m+n)}$ (b) $\beta(m, n) = \frac{\Gamma(m+n)}{\Gamma(m)\Gamma(n)}$
 (c) $\beta(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$ (d) $\beta(m, n) = \Gamma(m)\Gamma(n)\Gamma(m+n)$
- (ii) Which of the following functions contains odd terms in Taylor's series
 (a) $\cos x^2$ (b) $\sin x^2$ (c) $\sin x^3$ (d) $\cos x^3$
- (iii) If λ is the eigen value of a non-singular matrix A then eigen value of A^{-1} is
 (a) $\frac{1}{\lambda}$ (b) $\lambda-1$ (c) $1-\lambda$ (d) λ
- (iv) An $m \times n$ matrix of rank r can be expressed as a product of 2 matrices each of rank
 (a) $\frac{r}{2}$ (b) 2r (c) r-1 (d) r
- (v) Interchanges of two rows(or columns) of an $n \times n$ matrix A, changes
 (a) The sign of determinant A (b) The rank of matrix A
 (c) The eigen values of A (d) None of these
- (vi) The chance that a leap year selected at random will contain 53 Wednesdays is
 (a) $\frac{2}{7}$ (b) 0 (c) 1 (d) $\frac{6}{7}$
- (vii) If X is normally distributed with mean 0 and variance 1, then the expectation of X^2 is
 (a) 1 (b) 2 (c) 8 (d) 0
- (viii) For the p.d.f. $f(x) = \left(\frac{1}{4}\right)$, $-2 < x < 2$, then the value of E(X) is
 (a) 1 (b) -1 (c) 3 (d) 0
- (ix) A passing student is failed by an examiner, it is an example of
 (a) Type-I error (b) Type-II error
 (c) Best decision (d) All of the above

- (x) If T is an unbiased estimator of θ , then \sqrt{T} a biased estimator of
 (a) θ^2 (b) $\sqrt{\theta}$ (c) θ (d) All of the above

Fill in the blanks with the correct word

- (xi) Rolle's theorem is applicable for $f(x)$ in $[a,b]$ if _____.
 (xii) Every skew symmetric determinant of odd order is equal to _____.
 (xiii) The determinant of the matrix whose eigen values are 7, 1, 9 is given by _____.
 (xiv) Correlation coefficient between two uncorrelated variables is _____.
 (xv) The point estimator of population mean μ is _____.

Group - B

2. (a) State the geometric interpretation for "Lagrange MVT".
 If $f(h) = f(0) + hf'(\theta h), 0 < \theta < 1$.
 Show that the limiting value of θ as $h \rightarrow 0+$ is $\frac{1}{2}$ when $f(x) = \cos x$. [[CO1](Apply/IOCQ)]
 (b) Expand $f(x) = 5x^2 + 7x + 3$ in power of $(x-2)$ [[CO1](Evaluate/HOCQ)]
 (c) Find the value of ξ using MVT for $f(x) = x(x-1)(x-2)$ where $a=0, b=\frac{1}{2}$.
[[CO1](Analyze/IOCQ)]
4 + 4 + 4 = 12
3. (a) State a relation between Beta and Gamma function and show that
 $\int_0^1 x^{\frac{3}{2}} (1-x)^{\frac{3}{2}} = \frac{3\pi}{128}$. [[CO1](Analyze/IOCQ)]
 (b) Evaluate the following
 $\int_0^1 \frac{dx}{(1-x^6)^{\frac{1}{6}}}$ [[CO1](Execute/IOCQ)]
6 + 6 = 12

Group - C

4. (a) Let S be a subset of \mathbb{R}^3 defined by $S = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 = z^2\}$. Show that S is not a subspace of \mathbb{R}^3 . [[CO2](Apply/IOCQ)]
 (b) Summarise that intersection of two sub spaces of a vector space V over the field F is also a sub space of V . [[CO2](Understand/LOCQ)]
6 + 6 = 12
5. (a) Find the rank of the following matrix.
 (i) $\begin{pmatrix} 0 & 0 & 2 & 2 & 0 \\ 1 & 3 & 2 & 4 & 1 \\ 2 & 6 & 2 & 6 & 2 \\ 3 & 9 & 1 & 10 & 6 \end{pmatrix}$ (ii) $\begin{pmatrix} 4 & 2 & 6 & -1 \\ 6 & 1 & 3 & 8 \\ 16 & 4 & 12 & 15 \\ 10 & 3 & 9 & 7 \end{pmatrix}$ [[CO2](Evaluate/HOCQ)]

- (b) Show that the following is a linear transformation:
 $T: \mathbb{R}^3 \rightarrow \mathbb{R}^2$ is defined by $T(x, y, z) = (2x + y - z, x + y + z)$ [[CO2](Apply/IOCQ)]
 $(3 + 3) + (3 + 3) = 12$

Group - D

6. (a) There are $(2n + 1)$ tokens bearing numbers $1, 2, 3, \dots, (2n + 1)$ in a box. If 3 tokens are drawn at random from the box, then find the probability that the number on the chosen tokens are in arithmetic progression. [[CO4](Analyse/IOCQ)]
- (b) The probabilities of solving a problem by three students A, B and C are $\frac{3}{7}, \frac{3}{8}$ and $\frac{1}{3}$. If all of them try independently, find the probability that the problem could be solved by one student only. Find also the probability that the problem is not solved. [[CO4](Apply/IOCQ)]
 $6 + 6 = 12$

7. (a) The following Table - 1 shows the probability distribution of a random variable X:

Table - 1

X	0	1	2	3	4	5	6	7
P(X = x)	0	k	2k	2k	3k	k ²	2k ²	7k ² + k

- (i) Determine the constant k.
(ii) Evaluate $P(3 \leq X \leq 6)$ and $P(X > 3/X \leq 6)$ [[CO5](Analyse/IOCQ)]
- (b) Determine $f(x)$ the pmf of X from the following relation $f(x) = \frac{\lambda}{x} f(x-1): x = 1, 2, 3$ to ∞ . Also find $P(X > 0)$ [[CO5](Apply/LOCQ)]
 $(2 + 2 + 2) + 6 = 12$

Group - E

8. (a) If T_1, T_2 and T_3 are independent unbiased estimators of θ , and the variances of T_1, T_2 and T_3 are in the proportion of 2: 3 : 4, then which of the following would you prefer as a minimum variance unbiased estimator of θ ?

$$\frac{T_1 + T_2 + T_3}{3}, \quad \frac{2T_1 + T_2 + T_3}{4}, \quad \frac{3T_1 - T_2 + 2T_3}{4}$$

- (b) If $X_1, X_2, X_3, X_4, X_5, X_6$ are an independent simple random sample from a normal population with unknown variance σ^2 , find k, so that $k[(X_1 - X_2)^2 + (X_3 - X_4)^2 + (X_5 - X_6)^2]$ is an unbiased estimator of σ^2 . [[CO6](Understand/LOCQ)]
 $6 + 6 = 12$

9. (a) In order to test whether a coin is unbiased, the coin is tossed 5 times. The null hypothesis of unbiasedness of the coin is rejected if no head or 5 heads are

obtained. (i) What is the probability of Type I error? (ii) Find the probability of Type II error when the corresponding probability of head is 0.3. *[[CO6)(Analyse/IOCQ]]*

(b) A coin is tested for unbiasedness. The hypothesis that it is unbiased is rejected if 9 or more tosses of the coin out of 10 tosses result in head. Can we take 1% as the level of significance? *[[CO6)(Analyse/IOCQ]]*

6 + 6 = 12

Cognition Level	LOCQ	IOCQ	HOCQ
Percentage distribution	18.75	70.83	10.42