

**OPERATIONS RESEARCH
(MECH 4143)**

Time Allotted : 2½ hrs

Full Marks : 60

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 4 (four) from Group B to E, taking one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A

1. Answer any twelve:

12 × 1 = 12

Choose the correct alternative for the following

- (i) Activity which starts only after finishing other activity is called _____.
(a) Dummy (b) Successor
(c) Predecessor (d) Null
- (ii) Essential characteristics of a decision model are
(a) States of nature (b) Decision alternatives
(c) Payoff (d) All of these
- (iii) Simplex method of solving linear programming problem uses
(a) All the points in the feasible region
(b) Only the corner points of the feasible region
(c) Intermediate points within the infeasible region
(d) Only the interior points in the feasible region.
- (iv) A feasible solution to the linear programming problem should
(a) Satisfy the problem constraints
(b) Optimize the objective function
(c) Satisfy the problem constraints and non-negativity restrictions
(d) Satisfy the non-negativity restrictions
- (v) To find an optimum solution in transportation problems _____ method is used.
(a) Simplex (b) Big-M (c) MODI (d) Hungarian
- (vi) In an assignment problem having n facilities and n jobs, what is the number of possible ways of making assignments?
(a) n! (b) n² (c) 2n (d) 2ⁿ
- (vii) Hungarian algorithm is used to solve
(a) Transportation problem
(b) Assignment problem
(c) Unconstrained nonlinear programming problem
(d) Constrained nonlinear programming problem

- (viii) In a non-linear programming problem,
 (a) All the constraints should be linear
 (b) All the constraints should be non-linear
 (c) Either the objective function or at least one of the constraints should be non-linear
 (d) The objective function and all constraints should be linear.
- (ix) If the operating characteristics of a queue are dependent on time, then is said to be:
 (a) Transient state (b) Busy state
 (c) Steady-state (d) Explosive state
- (x) The queue discipline in stack of plates is:
 (a) LIFO (b) SIRO (c) Non-Pre-Emptive (d) FIFO

Fill in the blanks with the correct word

- (xi) In a LPP, the dual of a dual is _____.
- (xii) In a LPP, the objective function is always _____.
- (xiii) Service mechanism in a queuing system is characterized by _____ behaviours.
- (xiv) An activity which does not consume neither any resource nor time is known as _____ activity.
- (xv) The solution to a transportation problem with m-sources and n-destinations is feasible if the numbers of allocations are _____.

Group - B

2. (a) A factory manufactures three types of boxes. The fixed and variable costs are given below.

	Fixed Cost	Variable Cost
B ₁	20000	10
B ₂	30000	8
B ₃	50000	5

The likely demands under three situations are i. Poor demand: 2000 units, ii. Moderate demand: 5000 units and iii. High demand: 10000 units. If the sale price of each type is Rs. 20, prepare the payoff table. [[CO1](Understand/LOCQ)]

- (b) Identify how the decision under risk is different from decision making under condition of uncertainty. [[CO1](Understand/LOCQ)]

8 + 4 = 12

3. A dairy firm wants to determine the quantity of butter it should produce to meet the demand. Past records have shown the following demand patterns:

Qty. Reqd.	15	20	25	30	35	40	50
No. of Days	6	14	20	80	40	30	10

The stock levels are restricted to the range 15 to 50 kg due to inadequate storing facilities. Butter costs Rs. 40 per kg and is sold at Rs. 50 per kg.

- (i) Construct a conditional profit table.

(ii) Determine the action alternative associated with maximization of expected profit.

(iii) Determine EVPI.

[[CO1](Evaluate/HOCQ)]

(6 + 2 + 4) = 12

Group - C

4. A company manufactures two products X and Y. The profit contribution of X and Y are Rs. 3/- and Rs. 4/- respectively. The products X and Y require the services of three facilities. The capacities of the three facilities A, B, and C are limited and the available capacities in hours are 200 hrs, 150 hrs and 120 hrs. respectively. Product X requires 5, 3 and 4 hours of facilities A, B and C, respectively. Similarly, the requirement of product Y is 4, 5 and 3 hours respectively on A, B and C. Formulate the problem to find the optimal product mix to maximize the profit. Solve the problem using graphical method. Also construct the dual of the Problem.

[[CO3,CO4](Apply/IOCQ)]

(4 + 5 + 3) = 12

5. (a) Use Simplex Method to solve the following problem:

$$\text{Maximize } Z = 5x + 2y + 3z$$

Subject to -

$$x + 2y + 2z \leq 8$$

$$3x + 4y + z \leq 7$$

$$x, y, z \geq 0$$

[[CO4](Apply/IOCQ)]

- (b) Determine the initial basic feasible solution of the following transportation problem by Vogel's approximation method:

Factory	Warehouse				Capacity
	W ₁	W ₂	W ₃	W ₄	
F ₁	19	30	50	10	7
F ₂	70	30	40	60	9
F ₃	40	8	70	20	18
Requirement	5	8	7	14	34

[[CO5] (Apply / IOCQ)]

6 + 6 = 12

Group - D

6. (a) Show that the following function $f(x)$ is convex by expressing it as a sum of functions of one or two variables and then proving that all the functions are convex.

$$f(x) = 5x_1 + 2x_2^2 + x_3^2 - 3x_3x_4 + 4x_4^2 + 2x_5^4 + x_5^2 + 3x_5x_6 + 6x_6^2 + 3x_6x_7 + x_7^2$$

[[CO4] (Analyse/IOCQ)]

- (b) Verify whether the following function is convex or concave and find the maximum or minimum solution point.

$$f(x) = 4x_1^2 + 3x_2^2 + x_3^2 - 6x_1x_2 + x_1x_3 - 0.5x_1 - 2x_2 + 15.$$

[[CO4](Analyse/IOCQ)]

6 + 6 = 12

7. (a) Solve the NLPP utilizing Lagrange multiplier method.

$$\text{Maximize } Z = 4x_1 - x_1^2 + 8x_2 - x_2^2,$$

Subject to $x_1 + x_2 = 2$,

$x_1, x_2 \geq 0$.

[[CO4](Evaluate/HOCQ)]

(b) Solve the following NLPP graphically:

Maximize $Z = 10x_1 - x_1^2 + 10x_2 - x_2^2$,

Subject to $x_1 + x_2 \leq 12$,

$x_1 - x_2 \leq 6$,

$x_1, x_2 \geq 0$.

[[CO4](Evaluate/HOCQ)]

6 + 6 = 12

Group - E

8. (a) A person repairing radios finds that the time spent on the radio sets has exponential distribution with mean 20 minutes. If the radios are repaired in the order in which they come in and their arrival is approximately Poisson with an average rate of 15 for 8-hour day, what is the repairman's expected idle time each day? How many jobs are ahead of the average set just brought in? [[CO6](Analyse/IOCQ)]

(b) A bank has only one typist in a branch. The typing rate is randomly distributed approximating a Poisson distribution with mean service rate of 8 per hour. The letters arrive at a rate of 5 per hour during the entire 8 hour working day. If the typewriter is values at Rs. 150 per hour, determine

(i) Equipment utilization.

(ii) The percent of time that an arriving letter has to wait.

(iii) Average system time.

(iv) Average cost due to the idle time of the typewriter. [[CO6](Evaluate/HOCQ)]

6 + 6 = 12

9. Customers arrive at the first class ticket counter of a theatre at the range of 12 per hour. There is one clerk serving the customers at the rate of 30 per hour.

(i) What is the probability that there is no customer at the counter (i.e. the system is idle)?

(ii) What is the probability that there are more than 2 customers at the counter?

(iii) What is the probability that there are no customers at the counter waiting to be served?

(iv) What is the probability that a customer is being served and nobody is waiting? [[CO6](Analyse/IOCQ)]

12

Cognition Level	LOCQ	IOCQ	HOCQ
Percentage distribution	12.5	50	37.5