PHYSICS – II
(PHY2101)

Time Allotted : 2½ hrs **Full Marks : 60**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 4 (four) from Group B to E, taking one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A

1. Answer any twelve: **12 × 1 = 12**

Choose the correct alternative for the following

(i) The number of degrees of freedom of two particle connected by a rigid rod moving freely in a plane is
(a) 3 (b) 2 (c) 1 (d) 10

(ii) Both angular momentum and angular velocity will be parallel only if
(a) $I_{xx} = I_{yy} = I_{zz}$ (b) $I_{xx} \neq I_{yy} = I_{zz}$
(c) $I_{xx} = I_{yy} and\ I_{zz} = 0$ (d) $I_{xx} = I_{yy};\ I_{zz} \neq 0$

(iii) When the external force on a system of particles is withdrawn then _____ of the system remain constant,
(a) velocity of the system (b) velocity of the center of mass
(c) both (a) and (b) (d) none of (a) and (b)

(iv) The numbers of degrees of freedom of 3 particles moving in plane so that the distance between any two of them is always constant is
(a) 3 (b) 6 (c) 2 (d) 4

(v) The momentum corresponding to a cyclic coordinate is _____.
(a) function of t (b) function of q
(c) function of dq/dt (d) constant

(vi) For a system performing small oscillation around any point of equilibrium the potential energy matrix is
(a) positive definite (b) negative definite
(c) semidefinite (d) indefinite

(vii) If V_{jk} represents the potential energy matrix for system performing small oscillation then
(a) $Vjk = Vkj$ (b) $Vjk = -Vkj$
(c) $Vjk = Vkj^2$ (d) $Vjk = -Vkj^2$

(viii) Small oscillations happen around the point of _____ equilibrium.
(a) stable equilibrium (b) unstable equilibrium
(c) neutral equilibrium (d) metastable equilibrium

(ix) In an Eulerian velocity field, the fluid flow is steady when
(a) $\vec{\nabla}.\vec{V} = 0$ (b) $\vec{\nabla} \times \vec{V} = 0$
(c) $D\rho/Dt = 0$ (d) both (a) and (c)

(x) Streamline motions are found
(a) only in steady flows (b) only in unsteady flows
(c) both steady and unsteady flows (d) only in Newtonian fluids

Fill in the blanks with the correct word

(xi) Moment of Inertia of a sphere of radius 0.5 meters and radius 25 kg about an axis as diameter is _____.

(xii) The matrix representing inertia tensor is _____ in nature.

(xiii) The momentum corresponding to a cyclic coordinate is _____.

(xiv) Hamilton's equation of motion determines the trajectory in _____ space.

(xv) The differential form of the momentum equation for inviscid flows is _____ .

Group - B

2. (a) Define centre of mass of a rigid body having continuous mass distribution. Write down the mathematical expression of centre of mass for such system. *[(CO1)(Remember/LOCQ)]*

(b) Construct the coordinates of centre of mass for uniform solid hemisphere of radius 'a' and mass 'M'. *[(CO1)(Remember/LOCQ)]*

(c) Three particles of masses 2, 3 and 5 move under the influence of a force field so that their position vectors relative to a fixed coordinate system are given respectively by $\mathbf{r}_1 = 2t\,\mathbf{i} - 3\mathbf{j} + t^2\mathbf{k}$, $\mathbf{r}_2 = (t+1)\mathbf{i} + 3t\mathbf{j} - 4\mathbf{k}$ and $\mathbf{r}_3 = t^2\mathbf{i} - t\mathbf{j} + (2t-1)\mathbf{k}$ where t is the time. Compute the total angular momentum of the system. *[(CO1)(Apply/IOCQ)]*

4 + 4 + (2 + 2) = 12

3. (a) What do you mean by moment of inertia and product of inertia of a rigid body? When does the product of inertia zero? *[(CO1)(Understanding/LOCQ)]*

(b) Three equal masses are located at coordinates (a,0,0) ; (0,0,2a) ; (0,2a,a). Find the set of principal axes and the principal moment of inertia. *[(CO1)(Apply/IOCQ)]*

6 + 6 = 12

Group - C

4. (a) The Lagrangian of a system is given by $L = \frac{1}{2}\left(\dot{q}_1{}^2 + \dot{q}_2{}^2 + \dot{q}_1\dot{q}_2 - q_1\right)$ where symbols have their usual meaning.

(i) Find out the components of generalized momentum and identify the cyclic coordinate if any. *[(CO2)(Understand/LOCQ)]*

(ii) Find out the Lagrange equations of motion. *[(CO2)(Understand/LOCQ)]*

(iii) Construct the Hamiltonian of the system. *[(CO2)(Create/HOCQ)]*

(b) A simple pendulum of mass m and string length l is hanging from a fixed point of suspension. Write down the Cartesian coordinates of the system in terms of a suitable generalized coordinates. *[(CO2)(Understand/LOCQ)]*

(3 + 3 + 3) + 3 = 12

5. (a) Write down relation between action A and Lagrangian L of a typical variational problem. Show that the shape of a hanging chain between two points under constant gravity at its lowest potential energy is a catenary. *[(CO3)(Create/HOCQ)]*

(b) Write down the constraint relations of motion of a gas particle at any time t in an expanding spherical container of radius R. Identify the type of constraint. *[(CO2)(Understand/LOCQ)]*

(c) Show that if the Hamiltonian is not an explicit function of time it is a conserved quantity. *[(CO2)(Understand/LOCQ)]*

(2 + 4) + (2 + 1) + 3 = 12

Group - D

6. A particle of mass m is moving in a potential $V = (q)$ which is differentiable up to any arbitrary order at $q = 0$.

(i) If the particle performs a simple harmonic motion, show that the Lagrangian is given by $L = \frac{1}{2}m\dot{q}^2 - \frac{1}{2}\frac{d^2V}{dq^2}\Big|_{q=0} q^2$ *[(CO4)(Remember/LOCQ)]*

(ii) Construct the Lagrange equations of motion. *[(CO4)(Apply/IOCQ)]*

(iii) Show that the frequency is given by $\omega = \sqrt{\frac{1}{m}\frac{d^2V}{dq^2}\Big|_{q=0}}$ *[(CO4)(Remember/LOCQ)]*

(iv) Prove that such a system is conservative. *[(CO4)(Evaluate/HOCQ)]*

(4 + 2 + 2 + 4) = 12

7. A spring-mass system consisting of masses m_1 and m_2 connected with rigid supports with springs of spring constant k_1 and k_2 respectively. They are connected by a coupling spring of spring constant k_{12}.

(i) Find out the dimension of configuration space and phase space and construct the Lagrangian equations. *[(CO4)(Create/HOCQ)]*

(ii) Explain the conditions for which small oscillation is possible for the system. *[(CO4)(Understand/LOCQ)]*

(iii) Evaluate the normal frequencies of the system. Explain the symmetric and antisymmetric modes. *[(CO4)(Evaluate/HOCQ)]*

[(2 + 2) + 2 + (4 + 2)] = 12

Group - E

8. (a) Consider a fluid particle having an instantaneous velocity $\vec{V}(t)$. Develop the expression for the acceleration of the particle and express it in terms of the substantive derivative $\frac{D}{Dt}$. *[(CO5)(Create/HOCQ)]*

(b) The velocity field for a flow in the xy-plane of a gas is given by $u(x,y) = 4y/(x^2+y^2)$ and $v(x,y) = -4x/(x^2+y^2)$. Show that this is an incompressible flow. *[(CO5)(Remember/LOCQ)]*

(c) Develop the linear momentum equation for an inviscid fluid. *[(CO5)(Create/HOCQ)]*

(d) Show that $\frac{D\vec{V}}{Dt} = (\vec{V}.\vec{\nabla})\vec{V}$ assuming a steady flow. *[(CO5)(Remember/LOCQ)]*

3 + 3 + 4 + 2 = 12

9. (a) What is the difference between the Lagrangian and Eulerian descriptions of motion? Which description is, in general, used to study the mechanics of fluids and why? *[(CO5)(Remember/LOCQ)]*

(b) Develop the equation of continuity in differential form. *[(CO5)(Create/HOCQ)]*

(c) Consider an incompressible fluid in two-dimensional flow in the xy plane. If the x-component of the velocity field, $\vec{V}(t)$, is $u = 3\sinh x$, determine the expression for $\vec{V}(t)$ if its y-component at $y = 0$ is $v = \cosh x$. *[(CO5)(Evaluate/HOCQ)]*

(2 + 2) + 5 + 3 = 12

Cognition Level	LOCQ	IOCQ	HOCQ
Percentage distribution	47.9	12.5	39.6