ENGINEERING COMPUTATIONAL TECHNIQUES
(MECH 4124)

Time Allotted : 2½ hrs **Full Marks : 60**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and <u>any 4 (four)</u> from Group B to E, taking <u>one</u> from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A

1. Answer any twelve: **12 × 1 = 12**

Choose the correct alternative for the following

(i) Find the absolute error if the number $X = 0.00545828$ is truncated to three decimal digits is
(a) 0.45828×10^{-3} (b) 0.5828×10^{-4}
(c) 0.828×10^{-5} (d) None of the above

(ii) The method in which a new value of a variable, obtained by iteration, is used immediately in the following equations is called
(a) Gauss-Jordan method (b) Gauss-Seidel method
(c) Jacobi's method (d) Relaxation method

(iii) Which of the following is an iterative method?
(a) Gauss Seidel (b) Gauss Jordan
(c) Factorization (d) Gauss Elimination

(iv) If $f(x) = 0$ is an algebraic equation, the Newton-Raphson method for approximating the root of the equation is given by $\mathbf{x_{n+1} = x_n - f(x_n)/?}$:
(a) $f(x_{n-1})$ (b) $f'(x_{n-1})$
(c) $f'(x_n)$ (d) $f''(x_n)$

(v) The Newton-Gregory Forward Interpolation formula can be used
(a) Only for equally spaced intervals of argument values
(b) Only for unequally spaced intervals of argument values
(c) For both equally and unequally spaced intervals of argument values
(d) For unequal spacing of argument values.

(vi) A third-order forward difference can be expressed as
(a) $\Delta^3 y_0 = y_3 - 3y_2 + 3y_1 - y_0$ (b) $\Delta^3 y_0 = y_3 + 3y_2 + 3y_1 + y_0$
(c) $\Delta^3 y_0 = y_3 + 3y_2 - 3y_1 - y_0$ (d) $\Delta^3 y_0 = y_3 - 3y_2 - 3y_1 - y_0$

(vii) Which of the following methods is the best for solving initial value problems:
(a) Taylor's series method (b) Euler's method
(c) Modified Euler's method (d) Runge-Kutta method of the fourth order

(viii) Lagrange interpolating polynomial is the rearrangement of
(a) Newton's interpolating polynomials
(b) Newton's forward interpolating method
(c) Newton's backward interpolating method
(d) None of the above

(ix) An example of initial boundary value (IVP) problem governed by linear second order partial differential equation is
(a) $u_{xx} + u_{yy} = 0$ (b) $u_{xx} + u_{yy} = G(x, y)$
(c) $u_t = c^2 u_{xx}$ (d) Both (a) and (b)

(x) The Trapezoidal rule approximates the area under a curve as:
(a) A sum of rectangles (b) A sum of parabolic arcs
(c) A sum of trapezia (d) A sum of triangles

Fill in the blanks with the correct word

(xi) Out of the Regula-falsi method and the Newton-Raphson method, the rate of convergence is faster for ____________________

(xii) The finite difference scheme of the differential equation $y'' + 2y = 0$ is ____________

(xiii) The value of $\int_0^{0.5} \frac{dx}{1+x}$ by Simpson's 1/3 rule is ________________ (Divide the interval into two parts)

(xiv) The one-dimensional heat conduction equation $\frac{\partial T}{\partial t} = \alpha^2 \frac{\partial^2 T}{\partial x^2}$ is ________. (Classify the type of PDE)

(xv) In a partial differential equation, the order is determined by the ________ order of the partial derivatives appearing in it.

Group - B

2. (a) Write the expression of approximation error. Write down two differences between true error and approximated error. *[(CO1) (Understand/LOCQ)]*
(b) Find a root of the equation $x^3 - 4x - 9 = 0$ using the Bisection method correct to three decimal places. *[(CO2) (Apply/IOCQ)]*

(2 + 2) + 8 = 12

3. (a) Find the number of terms of the exponential series such that their sum gives the value of e^x correct to six decimal places at x = 1. *[(CO1) (Apply/IOCQ)]*
(b) Find a real root of the equation $x^3 - 2x - 5 = 0$ by the Regula-falsi method. Continue the calculations till the fifth approximation. *[(CO2) (Apply/IOCQ)]*

6 + 6 = 12

Group - C

4. (a) Solve by Jacobi's iteration method, the equations $10x + y - z = 11.19$; $x + 10y + z = 28.08$; $-x + y + 10z = 35.61$. Continue the computations till the fifth set of iterations. *[(CO2) (Apply/IOCQ)]*

(b) Fit a cubic polynomial which takes the following values:

x:	3	5	7	9
f(x):	6	24	60	120

Hence evaluate f (4). *[(CO3) (Apply/IOCQ)]*

7 + 5 = 12

5. (a) Apply the Gauss-Siedel iteration method to solve the equations $2x + y + z = 5$; $3x + 5y + 2z = 15$; $2x + y + 4z = 8$. Continue the computations till the fifth set of iterations. *[(CO2) (Apply/IOCQ)]*

(b) Find the eigenvalues and eigenvectors of the matrix $\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$. *[(CO3) (Apply/IOCQ)]*

7 + 5 = 12

Group - D

6. (a) Consider the following data:

x:	2	4	6	8
f(x):	3	5	6	7

With respect to the above table, evaluate the definite integral $\int_2^8 f(x)dx$ by using the Trapezoidal rule for numerical integration. *[(CO4) (Apply/IOCQ)]*

(b) Evaluate the definite integral $\int_0^5 \frac{dx}{(1+x^2)}$ by using (i) Trapezoidal rule and (ii) Simpson's 3/8 rule. Divide the interval (0, 5) into six parts each of width h = 1. *[(CO4) (Apply/IOCQ)]*

6 + 6 = 12

7. (a) Apply the Runge-Kutta fourth-order method to find an approximate value of y when x = 0.1, given that $dy/dx = xy + y^2$ and $y\ (x = 0) = 1$. Take h = 0.1. *[(CO5) (Apply/IOCQ)]*

(b) Apply the Runge-Kutta fourth-order method to find an approximate value of y when x = 0.1, given that $dy/dx = x - 2y$ and $y\ (x = 0) = 1$. Take h = 0.1. *[(CO5) (Apply/IOCQ)]*

6 + 6 = 12

Group - E

8. (a) Given the values of u(x, y) on the boundary of the square in the Figure below, evaluate the function u(x, y) satisfying the Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$at the pivotal points of this Figure by the Gauss-Seidel method. Show the steps clearly upto the fifth iteration.

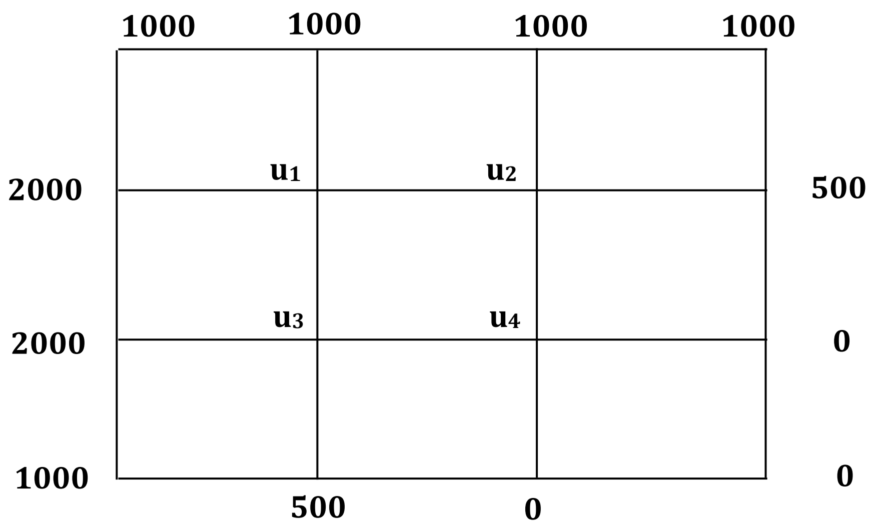

Figure: Refer to Problem 8 (a).

[(CO6) (Analyse/HOCQ)]

(b) Cite an example of an elliptic, second-order, linear partial differential equation.

[(CO6) (Understand/LOCQ)]

10 + 2 = 12

9. (a) Consider the ODE $\frac{d^2y}{dx^2} = 3x + 2y$ with the boundary conditions $y(0) = y(1) = 0$. Divide the interval [0,1] into four equal sub-intervals. With the help of the finite difference method, formulate the difference equations at each of these nodes.

[(CO5) (Apply/IOCQ)]

(b) Consider the ODE $\frac{d^2y}{dx^2} = x - 2y$ with the boundary conditions $y(0) = 0$; $y(1) = 2$. Using the finite difference method, find $y\,(0.25)$, $y\,(0.5)$, and $y\,(0.75)$ satisfying the differential equation. Dividing the interval (0, 1) into four equal sub-intervals.

[(CO5) (Analyse/HOCQ)]

6 + 6 = 12

Cognition Level	LOCQ	IOCQ	HOCQ
Percentage distribution	6.25	77.08	16.67