METHODS IN OPTIMIZATION
(MATH 4121)

Time Allotted : 2½ hrs **Full Marks : 60**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 4 (four) from Group B to E, taking one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A

1. Answer any twelve: **12 × 1 = 12**

Choose the correct alternative for the following

(i) The dual of a dual of an L.P.P. is
(a) primal (b) dual
(c) neither primal nor dual (d) always infeasible

(ii) If in the Big-M method, the set of basic variables of the final simplex table contains an artificial variable, the problem has
(a) degenerate solution (b) infeasible solution
(c) unbounded solution (d) multiple optimal solution

(iii) For a minimization type problem, if all $z_j - c_j \leq 0$, then the initial basic feasible solution is
(a) an optimum basic feasible solution (b) unbounded solution
(c) not an optimum basic feasible solution (d) infeasible solution

(iv) For a maximization problem, the objective function coefficient for an artificial variable is (where $M > 0$)
(a) $+M$ (b) $-M$ (c) 0 (d) $^1/_M$

(v) Matrix minima method is used for solving
(a) assignment problem (b) game theory
(c) non linear programming problem (d)transportation problem

(vi) If a constant be added to any row or any column of the cost matrix of an assignment problem, then
(a) the resulting assignment problem has the same optimal solution as the original problem
(b) the resulting assignment problem has a different optimal solution from the original problem
(c) the resulting assignment problem will not give an optimal solution
(d) the optimal solution of the resulting assignment problem can be obtained by adding same constant to the optimal solution of the original problem.

(vii) The range of values of p and q for which 2 is the value of the following game, is given by:

PLAYER B

PLAYER A

2	q	6
p	5	10
6	3	4

(a) $p \geq 5 \& q \geq 5$ (b) $p \geq 7 \& q \leq 5$
(c) for any value of p & q (d) $p \leq -5 \& q \geq 5$.

(viii) The bordered Hessian matrix of the Lagrangian function L constructed for an optimization problem with one equality constraint is given by

$$H^B(L) = \begin{pmatrix} 0 & 2 & 4 & 1 \\ 2 & 0 & 3 & 2 \\ 4 & 3 & 3 & 1 \\ 1 & 2 & 1 & 3 \end{pmatrix}$$

then the number of independent variables on which the objective function is depending on is,
(a) 1 (b) 2 (c) 3 (d) 0

(ix) If $(-1, 1)$ is a stationary point of the function $f(x, y)$ such that
$\frac{\partial^2 f}{\partial x^2} = x^2 + y^2, \frac{\partial^2 f}{\partial y^2} = x^2$ and $\frac{\partial^2 f}{\partial x \partial y} = xy$
then
(a) $(-1, 1)$is a local maximum point but not global
(b) $(-1, 1)$is a saddle point
(c) $(-1, 1)$is a local minimum point but not global
(d) $(-1, 1)$is a local minimum point.

(x) In Golden section search algorithm, if the initial interval of uncertainty is L_0, then the length of the final interval L_j is,
(a) $L_j = \frac{1}{\gamma^{j+2}} L_0$ (b) $L_j = \frac{1}{\gamma^j} L_0$ (c) $L_j = \frac{1}{\gamma^{j+1}} L_0$ (d) $L_j = \frac{1}{\gamma^{j-1}} L_0$
(where γ is the golden ratio)

Fill in the blanks with the correct word

(xi) While solving a maximization type LPP using Simplex algorithm, if at least one index number $z_j - c_j < 0$ the solution is ____________.

(xii) The algorithm used for solving linear programming problem is _______ algorithm.

(xiii) A feasible solution to a transportation problem is said to be _________ if and only if the corresponding cells in the transportation table do not contain a loop.

(xiv) In a _________ game gains of one player are equal to the losses of other player.

(xv) The function $f(x) = 2x^3 - 3x^2$ is convex, for all __________. (write the rage of x)

Group - B

2. (a) Solve the following L.P.P. by Simplex method:
Maximize $z = 5x_1 + 3x_2$
subject to the constraints

$$x_1 + x_2 \leq 2$$
$$5x_1 + 2x_2 \leq 10$$
$$3x_1 + 8x_2 \leq 12$$
$$x_1,\ x_2 \geq 0.$$

[(MATH4121.1, MATH4121.2) (Apply/IOCQ)]

(b) Find the dual of the following L.P.P.:

Maximize $z = x_1 + 4x_2 + 3x_3$

subject to

$$2x_1 + 3x_2 - 5x_3 \leq 2$$
$$3x_1 - x_2 + 6x_3 \geq 1$$
$$x_1 + x_2 + x_3 = 4$$
$x_1,\ x_2 \geq 0;\ x_3$ is unrestricted in sign.

[(MATH4121.1, MATH4121.2)(Understand/LOCQ)]

7 + 5 = 12

3. (a) A company makes two kinds of leather belts A *and* B. Their respective unit profits are $Rs. 4$ and Rs. 3 . One belt of type A requires 2 hours and one belt of type B requires 1 hour of time in making. The total man-hours available are 1000 per day. Due to insufficient supply of leather, the company can make only 800 belts per day. Only 400 buckles for type A and 700 buckles of type B are available. Formulate the problem as an LPP and solve it graphically. *[(MATH4121.1, MATH4121.2)(Create/HOCQ)]*

(b) Solve the following L.P.P. by Big-M method:

Minimize $z = 12x_1 + 20x_2$

subject to the constraints

$$6x_1 + 8x_2 \geq 100$$
$$7x_1 + 12x_2 \geq 120$$
$$x_1,\ x_2 \geq 0$$

[(MATH4121.1, MATH4121.2)(Apply/IOCQ)]

5 + 7 = 12

Group - C

4. (a) Find the optimal solution of the following transportation problem:

	D_1	D_2	D_3	D_4	Supply
O_1	2	2	2	1	3
O_2	10	8	5	4	7
O_3	7	6	6	8	5
Demand	4	3	4	4	

[(MATH4121.1, MATH4121.2, MATH4121.3, MATH4121.4)(Evaluate/HOCQ)]

(b) Find the optimal assignment from the following profit matrix:

	1	*2*	*3*	*4*
A	40	35	43	45
B	33	39	48	33
C	40	37	33	32
D	35	41	39	37

[(MATH4121.1, MATH4121.2 MATH4121.3, MATH4121.4)(Evaluate/HOCQ)]

7 + 5 = 12

5. (a) Use algebraic method to solve the following game:

PLAYER B

PLAYER A

9	1	4
0	6	3
5	2	8

[(MATH4121.1, MATH4121.2, MATH4121.3, MATH4121.4)(Apply/IOCQ)]

(b) Use graphical method in solving the following game and find the value of the game:

PLAYER B

PLAYER A

−6	−1	4	3
7	−2	−5	7

[(MATH4121.1, MATH4121.2, MATH4121.3, MATH4121.4) (Apply/IOCQ)]

6 + 6 = 12

Group - D

6. (a) Use the method of Lagrange's multipliers to solve the following non- linear programming problem. Does the solution maximize or minimize the objective function?

Optimize $Z = 2x_1^2 + x_2^2 + 3x_3^2 + 10x_1 + 8x_2 + 6x_3 - 100$

Subject to the constraint

$$x_1 + x_2 + x_3 = 20$$

[(MATH4121.6)(Evaluate/HOCQ)]

(b) Verify whether the following function is convex or concave and find the maximum or minimum solution point:

$f(x_1, x_2, x_3) = 4x_1^2 + 3x_2^2 + x_3^2 - 6x_1x_2 + x_1x_3 - \frac{x_1}{2} - 2x_2 + 15.$

[(MATH4121.6)(Understand/LOCQ)]

8 + 4 = 12

7. (a) Use Kuhn-Tucker conditions to solve the following non-linear programming problem:

Minimize $Z = x_1^2 + x_2^2 - 2x_1 - 4x_2$

subject to the constraints

$$x_1 + 4x_2 \leq 5$$
$$2x_1 + 3x_2 \leq 6$$
$$x_1 \quad , x_2 \geq 0.$$

[(MATH4121.6)(Evaluate/HOCQ)]

(b) Determine the relative maximum and minimum (if any) of the following function:

$$f(x_1, x_2, x_3) = x_1^2 + x_2^2 + x_3^2 - 4x_1 - 8x_2 - 12x_3 + 56.$$

[(MATH4121.6)(Understand/LOCQ)]

8 + 4 = 12

Group - E

8. Use interval halving method to minimize $f(x) = x^4 - 12x^2 - 60x$ over [0, 2] taking the tolerance to be less than 0.3.

[(MATH4121.1, MATH4121.5) (Apply/IOCQ)]

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9. Use Fibonacci search algorithm to maximize $f(x) = x^4 - 14x^3 + 60x^2 - 70x$ in [0, 2] using 6 functional evaluations.

[(MATH4121.1, MATH4121.5) (Apply/IOCQ)]

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Cognition Level	LOCQ	IOCQ	HOCQ
Percentage distribution	13.5	52.1	34.4