

**PROBABILITY AND STATISTICS  
(MTH2102)**

**Time Allotted : 2½ hrs**

**Full Marks : 60**

*Figures out of the right margin indicate full marks.*

*Candidates are required to answer Group A and any 4 (four) from Group B to E, taking one from each group.*

*Candidates are required to give answer in their own words as far as practicable.*

**Group – A**

1. Answer any twelve:

**12 × 1 = 12**

*Choose the correct alternative for the following*

- (i) The distribution for which mean and variance are equal is  
(a) Normal (b) Binomial (c) Exponential (d) Poisson.
- (ii) If  $X$  has exponential distribution with parameter  $\lambda = \frac{1}{5}$ , then its standard deviation is  
(a) 25 (b)  $\frac{1}{5}$  (c)  $\frac{1}{25}$  (d) 5.
- (iii) If  $X$  and  $Y$  are two independent random variables and  $Z = X + Y$  then the moment generating function of  $Z$  i.e.  $M_Z(t)$  is  
(a)  $M_X(t)M_Y(t)$  (b)  $M_X(t) + M_Y(t)$   
(c)  $M_x(t)$  (d)  $M_Y(t)$ .
- (iv) If for a random variable  $X$ ,  $E\{(X - 2)^2\} = 6$  and  $E\{(X - 1)^2\} = 10$  then  $\text{Var}(X)$  is  
(a)  $\frac{7}{2}$  (b)  $\frac{15}{4}$   
(c) 0 (d)  $\sqrt{2}$ .
- (v) If the joint probability density function of the random variable  $X$  and  $Y$  is given by  
$$f(x) = \begin{cases} Cx(1 - x), & 0 \leq x \leq y \leq 1 \\ 0, & \text{otherwise} \end{cases}$$
, then the value of  $C$  is  
(a) 1 (b) 2  
(c) 3 (d) 4.
- (vi) For normal distribution, the coefficient of skewness ( $\gamma_1$ ) and kurtosis ( $\gamma_2$ ) are  
(a)  $\gamma_1 = 1, \gamma_2 = 0$  (b)  $\gamma_1 = 0, \gamma_2 = 3$   
(c)  $\gamma_1 = 0, \gamma_2 = 0$  (d)  $\gamma_1 = 3, \gamma_2 = -3$ .
- (vii) The maximum and the minimum values for correlation coefficient between  $x$  and  $y$  are respectively  
(a) 1, 0 (b) 2, 1  
(c) 0, -1 (d) 1, -1.

- (viii) Two variables  $x$  and  $y$  are such that  $cov(x, y) = 0.32$ ,  $\sigma_x = 0.2$ ,  $\sigma_y = 0.4$ , then the regression coefficient of  $x$  on  $y$  is  
 (a) 2 (b) 1  
 (c) 0.8 (d) 0.6.
- (ix) If  $X, Y$  and  $Z$  are three independent standard normal variates, then  $X^2 + Y^2 + Z^2$  is a  
 (a) standard normal variate  
 (b)  $\chi^2$  - variate with degrees of freedom 3  
 (c)  $\chi^2$  - variate with degrees of freedom 2  
 (d)  $\chi^2$  - variate with degrees of freedom 1.
- (x) The maximum likelihood estimate is a solution of the equation  
 (a)  $\frac{\partial}{\partial \theta} L(\theta) = 0$  (b)  $\frac{\partial}{\partial \theta} L(\theta) = \text{constant}$   
 (c)  $\frac{\partial}{\partial \theta} L(\theta) = \theta$  (d)  $\frac{\partial}{\partial \theta} L(\theta) = 1$ .

*Fill in the blanks with the correct word*

- (xi) If  $X$  has mean 4,  $Y$  has mean  $-2$ , then mean of  $2X + Y - 5$  is \_\_\_\_\_.
- (xii) The mean and standard deviation of a Binomial distribution are respectively 4 and  $\frac{\sqrt{8}}{3}$ , then the values of  $n$  and  $p$  (where  $n$  and  $p$  are the parameters) are respectively \_\_\_\_\_.
- (xiii) The median for the given data: 10, 5, 9, 4, 8, 7, 6 is \_\_\_\_\_.
- (xiv) For a 99% confidence interval, the confidence coefficient is \_\_\_\_\_.
- (xv) In a Markov chain, the sum of transition probabilities from a given state equals to \_\_\_\_\_.

### Group - B

2. (a) The life length  $X$  of an electronic component follows an exponential distribution. There are two processes by which the component may be manufactured. The expected life length of the component is 100 hours if process-I is used to manufacture, while it is 150 hours if process-II is used. The cost of manufacturing a single component by process-I is Rs. 10 while it is Rs. 20 for process-II. Moreover, if the component lasts less than the guaranteed life of 200 hours, a loss of Rs 50 is to be borne by the manufacturer. Which process is advantageous to the manufacturer?  
 [(MTH2102.1, MTH2102.2)(Apply/IOCQ)]
- (b) Find the moment generating function of the Exponential distribution. Hence use it to find mean and variance.  
 [(MTH2102.1, MTH2102.2)(Apply/IOCQ)]  
**6 + 6 = 12**
3. (a) Show by Chebyshev's inequality that 2000 throws with a coin, the probability that the number of heads lies between 900 and 1100 is at least  $19/20$ .  
 [(MTH2102.1, MTH2102.2) (Apply/IOCQ)]
- (b) A fair coin is tossed 400 times. Using normal approximation to binomial distribution find the probability of obtaining (i) exactly 200 heads, (ii) between 190 and 210 heads, both inclusive.  
 [(MTH2102.1, MTH2102.2) (Apply/IOCQ)]  
**6 + 6 = 12**

### Group - C

4. (a) The random variables  $X$  and  $Y$  have a joint probability mass function given by  $P(X = x, Y = y) = \frac{x^2 + y^2}{32}$  for  $x = 0, 1, 2, 3$  and  $y = 0, 1$ . Find the marginal probability mass function of  $X$  and  $Y$ . Also find  $P(X \leq 2, Y = 1)$ .  
[(MTH2102.1, MTH2102.2, MTH2102.3) (Apply/IOCQ)]
- (b) An urn always contains two balls of colours red and blue. At each stage a ball is randomly chosen and then replaced by a new ball, which with probability 0.8 in case of same colour replacement and with probability 0.2 in case of opposite colour replacement. If initially both the balls are red, find the probability that the 3<sup>rd</sup> ball selected is red.  
[(MTH2102.1, MTH2102.2, MTH2102.3) (Understand/LOCQ)]
- 6 + 6 = 12**

5. (a) If  $X$  and  $Y$  have a joint p.d.f.:  $f(x, y) = \begin{cases} k, & \text{for } x^2 + y^2 \leq 4 \\ 0, & \text{elsewhere} \end{cases}$ , where  $k$  is a constant. Find the value of  $k$  and  $P(X^2 + Y^2 > 1)$ . Also find the marginal p.d.f. of  $Y$ .  
[(MTH2102.1, MTH2102.2, MTH2102.3) (Apply/IOCQ)]
- (b) The three state Markov chain is given by the transition probability matrix  $P = \begin{pmatrix} 0 & 2/3 & 1/3 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \end{pmatrix}$ . Prove that the chain is irreducible and all the states are aperiodic and non-null recurrent. Find also the steady state distribution of the chain.  
[(MTH2102.1, MTH2102.2, MTH2102.3) (Evaluate/HOCQ)]
- 6 + 6 = 12**

### Group - D

6. (a) The following table provides data about the percentage of students who have free university meal and their CGPA scores. Calculate the Spearman's Rank Correlation between the two and interpret the result.

State University	Pune	Chennai	Delhi	Kanpur	Goa	Indore	Guwahati
% of students having free meals	14.4	7.2	27.5	33.8	38.0	15.9	4.9
% of students scoring above 8.5 CGPA	54	64	44	32	37	68	62

[(MTH2102.3, MTH2102.4, MTH2102.5, MTH2102.6) (Apply/IOCQ)]

- (b) The expenditure of 1000 families is given below:

Expenditure (Rs)	40 – 59	60 – 79	80 – 99	100 – 119	120 – 139
Frequency	50	–	500	–	50

If the median and the mean for the distribution are both Rs. 87.50, then calculate the missing frequencies.  
[(MTH2102.3, MTH2102.4, MTH2102.5, MTH2102.6) (Apply/IOCQ)]

**6 + 6 = 12**

7. (a) Out of the two regression lines given by  $3x + 12y = 19$  and  $3y + 9x = 46$ , which one is the regression line of "x on y"? Justify your answer. Find the mean of  $x$  and mean of  $y$ . Find the correlation coefficient between  $x$  and  $y$ .  
[(MTH2102.3, MTH2102.4, MTH2102.5, MTH2102.6) (Analyze/IOCQ)]

(b) Compute the arithmetic mean and standard deviation for the following data:

Score:	4 – 5	6 – 7	8 – 9	10 – 11	12 – 13	14 – 15
Frequency:	4	10	20	15	8	3

[(MTH2102.3, MTH2102.4, MTH2102.5, MTH2102.6) (Remember/LOCQ)]

**6 + 6 = 12**

### Group - E

8. (a) A machine is supposed to fill bottles with 500 ml of juice. The standard deviation is known to be 10 ml. A random sample of size 25 bottles is taken. Suppose that the hypothesis  $H_0: \mu = 500$  is rejected in favour of  $H_1: \mu < 500$  if the sample mean is less than 496.7 ml. Find the probability of Type I error at this decision rule.

[(MTH2102.4, MTH2102.5, MTH2102.6)(Apply/IOCQ)]

(b) (i) If  $T_1$  and  $T_2$  be two statistics with expectation  $E(T_1) = 2\theta_1 + 3\theta_2$  and  $E(T_2) = \theta_1 + \theta_2$ , find unbiased estimators of the parameters  $\theta_1$  and  $\theta_2$ .

(ii) If the sample observations are 2, 4, 6, 8, 10 from an infinite population with variance  $\sigma^2$ , determine an unbiased estimate of  $\sigma^2$ .

[(MTH2102.4, MTH2102.5, MTH2102.6) (Apply/IOCQ)]

**6 + 3 + 3 = 12**

9. (a) The random variable  $X$  is normally distributed with mean 68 cm and standard deviation 2.5cm. What should be the size of the sample whose mean shall not differ from the population mean by more than 1cm with probability 0.95?

[(MTH2102.4, MTH2102.5, MTH2102.6) (Apply/IOCQ)]

(b) Let  $p$  denotes the probability of getting a head when a given coin is tossed once. Suppose that the hypothesis  $H_0: p = 0.5$  is rejected in favour of  $H_1: p = 0.7$  if 9 trials result in 6 or more heads. Calculate the probabilities of Type-I and Type-II error.

[(MTH2102.4, MTH2102.5, MTH2102.6) (Evaluate/HOCQ)]

**6 + 6 = 12**

Cognition Level	LOCQ	IOCQ	HOCQ
Percentage distribution	12.5	75	12.5