DATA STRUCTURES AND ALGORITHMS
(CSE2101)

Time Allotted : 2½ hrs **Full Marks : 60**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 4 (four) from Group B to E, taking one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A

1. Answer any twelve: **12 × 1 = 12**

Choose the correct alternative for the following

(i) Let A be a two-dimensional array declared as follows:
A: array [1…10] [1…15] of character;
Assuming that each character takes one memory location, the array is stored in row-major order and the first element of the array is stored in location 100, what is the address of the element A[i][j] ?
(a) 15i + j + 84 (b) 15j + i + 84
(c) 10i + j + 89 (d) 10j + i + 89

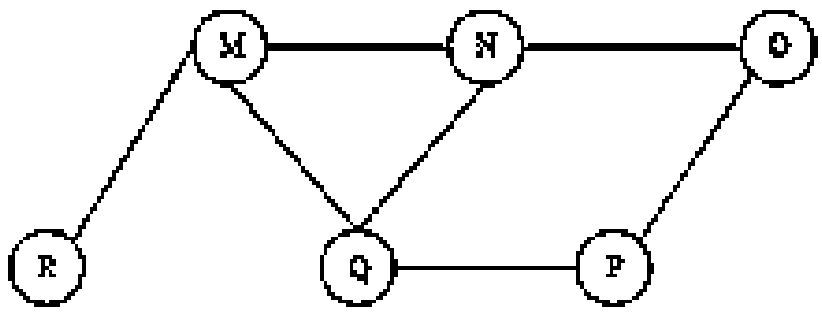
(ii) In the worst case, the number of comparisons needed to search for a key in a singly linked list of length n is:
(a) $\log_2 n$ (b) n/2 (c) $\log_2 n - 1$ (d) n

(iii) Which of the following statements is correct for a circular singly linked list with only a start pointer?
(a) Both insertion and deletion at the front end take O(1) time
(b) Only insertion at the front end takes O(1) time
(c) Only deletion from the front end takes O(1) time
(d) No insertion or deletion operation at either end is possible in O(1) time

(iv) What is the data structure used to implement the BFS traversal of a graph?
(a) Stack (b) Queue (c) Binary Tree (d) B-tree

(v) When using a stack to convert an infix expression into its postfix equivalent, which of the following is true?
(a) The stack may contain both opening and closing brackets
(b) A plus operator can be pushed on top of a binary minus operator
(c) A division operator can be pushed on top of a plus operator
(d) An open bracket can be popped off the stack when a division operator is processed

(vi) The Breadth First Search algorithm has been implemented using the queue data structure. One possible order of visiting the nodes of the following graph is



(a) MNOPQR (b) NQMPOR (c) QMNPRO (d) QMNPOR

(vii) Which of the following tree can always be stored with optimum space complexity, using a 1D array?
(a) Full Binary Tree (b) Almost complete Binary Tree
(c) Binary Search Tree (d) AVL tree.

(viii) In a graph G with V vertices and E edges, what is the time complexity of detecting a cycle using DFS traversal?
(a) O(V) (b) O(E) (c) O(V + E) (d) O(V × E)

(ix) When does the worst case complexity happen for Insertion Sort?
(a) When the array is already sorted
(b) When the array is sorted in reverse order
(c) When the array is not sorted
(d) When the array is sorted till the midpoint and then reverse sorted

(x) In an open hashing setup, suppose there are n elements and D buckets, the average case time complexity for insertion in the hash table:
(a) O(n) (b) O(1 + n/D) (c) O(1 – n/D) (d) $O(D \log_2 n)$

Fill in the blanks with the correct word

(xi) A linked list having n nodes (n >= 1) where no node stores a NULL pointer is a _____ linked list.

(xii) The optimal data structure used to solve the Tower of Hanoi problem is _____.

(xiii) For any B-tree of minimum degree t ≥ 2, every node other than the root can have at most _____ keys.

(xiv) If the in-order traversal of a tree is E A C K F H D B G, its pre-order traversal is _____.

(xv) If f(n)= O(g(n)) and g(n)=O(h(n)), then the relationship between f(n) and h(n) in terms of Big Oh notation is _____.

Group - B

2. (a) Prove: $n^2 + 100\,n = \theta(n^2)$. *[(CO1)(Understand & Remember/LOCQ)]*

(b) Write the pseudo-code of inserting an element after a given value in a doubly linked list. Your code should work for any position of the given value. Show proper error messages, if any. *[(CO1)(Understand & Remember/LOCQ)]*

(c) What is a sparse matrix?
Given the following 4 × 5 matrix:
0 0 6 0 0
0 0 0 0 0
4 0 2 0 0
0 3 0 7 0
Compute its triplet array equivalent.
Is it beneficial to store this matrix as a triplet array? Justify your answer. *[(CO1)(Compute/IOCQ)]*

4 + 4 + (1 + 1 + 2) = 12

3. (a) Write a function which will take any number n as its argument. The function will break this number into its individual digits and then store every single digit in a separate node thereby forming a linked list. The function must return the head node address of the created linked list at the end. (For example, if the number is 13579, then there will be 5 nodes in the list containing nodes with values 1, 3, 5, 7, 9). *[(CO5)(Apply/HOCQ)]*

(b) How can you represent the polynomial $5x^5 + 4x^2 - 25x + 10$ with array(s)? Additionally, if a singly linked list is used to represent this polynomial instead, would the arithmetic operations like addition/subtraction become more efficient? Provide your analysis. *[(CO2,CO5)(Apply)/IOCQ)]*

7 + (2 + 3) = 12

Group - C

4. (a) Write an algorithm to merge eight sorted queues into one single queue in such a way that duplicates are removed and the resultant queue is sorted also. Comment on the time complexity of your algorithm. More credits will be given to more efficient algorithms. *[(CO2)(Analyse/IOCQ)]*

(b) Convert the Infix expression ((A + B) * C – (D – E) ^ (F + G)) to equivalent Prefix and Postfix notations using stack. *[(CO2)(Apply/IOCQ)]*

(c) Write algorithm for inserting into an input restricted deque. Show the working of this algorithm by the input sequence: 1, 2 ,3 ,4, 5, 6. *[(CO2)(Remember/LOCQ)]*

4 + 4 + (2 +2) = 12

5. (a) There are n elements in a stack, stored in a sorted way. Move these elements to another stack so that the final stack is always sorted during the entire movement. You may additionally use another stack but the sorted property has to be maintained there all the time. An iterative version will fetch you additional credits. *[(CO2)(Analyse/IOCQ)]*

(b) Suppose you want to implement a queue and a stack using an unrestricted deque. Describe the deque methods that need to be reused to implement standard stack and queue operations. *[(CO2)(Apply/IOCQ)]*

(c) Explain what will be the output when fibo(5) is called:

```
void fibo(int n) {
    if (n==0) or (n==1) return;
    printf("Inside fibo(%d)\n",n);
    fibo(n-1);
    fibo(n-2);
    return;
}
```

[(CO2)(Apply/IOCQ)]

4 + 4 + 4 = 12

Group - D

6. (a) Briefly explain the difference between a full binary tree and a complete binary tree with suitable examples. *[(CO1, CO2)(Understand/LOCQ)]*

(b) Consider a binary search tree with **n** (i.e., **n >= 0**) nodes, which has been constructed using linked representation of tree, where the structure of each tree node has three components: key value/ data, pointer to left subtree and pointer to right subtree.

Suggest an algorithm (/ pseudo code/ C-code) of a function (or procedure) which will delete a given key value from the binary search tree, provided it is found in the tree *[(CO2, CO3, CO5)(Apply/IOCQ)]*

(c) Describe the outline of an algorithm for topological sort on a DAG which operates by repeatedly finding a vertex of zero in-degree and removing all its outgoing edges. Assume an adjacency matrix representation. Analyze the running time of your algorithm. Is your algorithm faster than one using DFS? *[(CO2 ,CO3 ,CO4 ,CO5)(Apply/IOCQ)]*

3 + 5 + 4 = 12

7. (a) Define a B-Tree of order m. *[(CO1)(Remember/LOCQ)]*

(b) (i)Show each step of insertions (along with any height balancing steps, if required), to construct an AVL tree using the keys, inserted one at a time, as per the following given order:
45, 80, 70, 40, 30, 65, 35, 25, 27, 15.
(ii) Show each steps of deletion (along with any height balancing steps, if required) from the above constructed AVL tree, where each of the following keys to be deleted (one at a time): 70, 80. *[(CO2, CO3)(Apply/IOCQ)]*

(c) You are trapped in a network of tunnels with only one exit. The network is connected in that all points are reachable. You have a box of chalk pieces to mark on the walls. Explain how you can come out of the tunnel network by searching for the exit efficiently. Analyze the running time of your algorithm. *[(CO2, CO3, CO6)(Apply/HOCQ)]*

2 + (5 + 1) + 4 = 12

Group - E

8. (a) (i) What is the expression for Load factor (α) in the context of Hashing by Open Addressing? Also, write what the terms signify that you use in the expression for α.
The average cost for linear hashing in case of –
Insertion is $1/2(1 + 1/(1 - \alpha)^2)$ and deletion is $1/2(1 + 1/(1 - \alpha))$
(ii) Which one do you think requires more time – insertion or deletion?
(iii) Show that your above answer follows from the above two expressions.
(iv) What could be the rationale behind it? *[(CO4, CO3)(Analyse/HOCQ)]*

(b) Fill up the following table with the appropriate asymptotic complexities:

Type of Sorting	Best Case	Average Case	Worst Case
Naïve Bubble Sort	$O(n^2)$		
Improved Bubble Sort			
Insertion Sort			
Selection Sort			$O(n^2)$
Counting Sort		$O(n + k)$	

[(CO4)(Remember/LOCQ)]

(1 + 1 + 2 + 2) + 6 = 12

9. (a) Suppose you want to sort the whole population of West Bengal, which is about 90 millions,in terms of their age in months. If you are thinking about run-time efficiency, which sorting algorithm would be most suitable and why? *[(CO2)(Select/IOCQ)]*

(b) Can you give a suitable example array with 10 integers on which interpolation search will have the same worst-case time complexity as linear search. *[(CO2)(Understand/LOCQ)]*

(c) Suppose you are given an array containing 0s and 1s in the following manner – a string of 0s trapped between two strings of 1s, eg. 111111000001111. It is also given that none of these three strings is empty, i.e., each of these three strings contains at least one character (or digit) and that the median (middle) element is a 0.

(i) Write a pseudo-code to determine both the left and right boundaries of the middle string of 0s, i.e. the output of your code should give the two indices of the leftmost 0 and the rightmost position 0.

(ii) What is minimum size of the array possible? *[(CO4)(Apply/IOCQ)]*

(1 + 2) + 2 + (6 + 1) = 12

Cognition Level	LOCQ	IOCQ	HOCQ
Percentage distribution	29.17	53.12	17.71