INFORMATION THEORY AND CODING
(CBS2102)

Time Allotted : 2½ hrs **Full Marks : 60**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 4 (four) from Group B to E, taking one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A

1. Answer any twelve: **12 × 1 = 12**

Choose the correct alternative for the following

(i) The capacity of a Binary Symmetric Channel (BSC) with cross probability 0.5 is
(a) 1 (b) 0 (c) 2 (d) 1.5

(ii) In the polynomial division algorithm for cyclic codes, what is the result of dividing a message polynomial by the generator polynomial?
(a) A quotient polynomial and a remainder polynomial
(b) A quotient polynomial and a parity check matrix
(c) An error vector and a corrected polynomial
(d) A syndrome vector and a corrected codeword

(iii) In GF(5), the additive group integer set is {0, 1, 2, 3, 4}. The inverse of element 4 is _____.
(a) 4 (b) 3 (c) 2 (d) 1

(iv) In a convolutional code, the term "code rate" is defined as:
(a) The ratio of the number of states to the number of memory elements
(b) The ratio of output bits to total bits in the codeword
(c) The ratio of input bits to output bits
(d) The ratio of the minimum distance to the constraint length

(v) What is the mutual information of a channel with independent inputs?
(a) Zero (b) Constant (c) Variable (d) Infinity

(vi) If a BCH code is used with a field size of 2^4 and designed to correct up to 2 errors, how many bits can the codeword have at most?
(a) 15 bits (b) 63 bits (c) 7 bits (d) 31 bits

(vii) For a (2, 1, 2) convolution encoder, impulse response is 1101. The output code word for data 101 is _____.
(a) 01110111 (b) 11011101
(c) 11001110 (d) 01010101

(viii) Basically, Galois field consists of _____ number of elements.
(a) finite (b) infinite
(c) both (a) and (b) (d) none of the above.

(ix) For a linear block code, which matrix represents the relationship between the codewords and the data bits?
(a) The generator matrix G (b) The parity check matrix H
(c) The syndrome matrix S (d) The error vector E

(x) If I(x1) and I(x2) are the information carried by two symbols x1 and x2 respectively then information carried compositely by x1 and x2 is _____.
(a) I(x1, x2) = I(x1) + I(x2) (b) I(x1, x2) = I(x1) - I(x2)
(c) I(x1, x2) = I(x1) * I(x2) (d) none of these.

Fill in the blanks with the correct word

(xi) The generator polynomial g(x) of (n,k) cycle code is a factor of ____.

(xii) A (n, k) block code consists of _____ number of check bits added to k number of information bits.

(xiii) The length of the output for a (2, 1, 3) convolution code for the message 1101 is _____.

(xiv) For the generation of a cyclic code, the generator polynomial should be the factor of _____.

(xv) The coding efficiency is expressed as _____.

Group - B

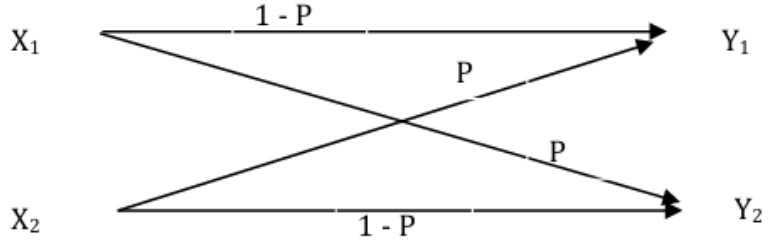
2. (a) Prove that the upper bound on entropy is given as $H_{Max} \leq \log_2 M$. Here M is the number of message emitted by the source. *[(CO3)(Evaluate/HOCQ)]*

(b) Prove that: $H(S^n) = nH(S)$, $H(S^n)$ is the higher order entropy of the source. *[(CO1)(Analyse/HOCQ)]*

(c) Prove that $I(X; Y) = H(X) - H(X|Y)$ *[(CO1)(Analyse/HOCQ)]*

5 + 4 + 3 = 12

3. (a) Consider a BSC in figure below with P(X1) = k.



(i) Prove that the mutual information $I(X; Y) = H(Y) + P \log_2 P + (1 - P) \log_2 (1 - P)$.
(ii) Compute I(X; Y) for k = 0.5 and P = 0.1. *[(CO1)(Apply/IOCQ)]*

(b) For a channel whose matrix is given below:

$$P(Y \mid X) = \begin{bmatrix} 0.6 & 0.2 & 0.2 \\ 0.2 & 0.6 & 0.2 \\ 0.2 & 0.2 & 0.6 \end{bmatrix}$$

Find I (X; Y) and channel capacity, given that input symbols occur with equal probability. *[(CO1)(Apply/IOCQ)]*

(4 + 2) + (3 + 3) = 12

Group - C

4. (a) What do you mean by hamming distance and burst length? *[(CO4)(Remember/LOCQ)]*

   (b) In an LBC, the syndrome is given by

   S1 = r1 + r2 + r3 + r5

   S2 = r1 + r2 + r4 + r6

   S3 = r1 + r3 + r4 + r7

   (i) Find the parity check matrix.

   (ii) Draw the encoder circuit.

   (iii) How many errors it can detect and correct?

   (iv) What is the syndrome for the received codeword 1011011? *[(CO4)(Apply/IOCQ)]*

2 + (2 + 2 + 3 + 3) = 12

5. Select the appropriate combination of (n, k) of the cyclic code generated by the following generator polynomials where $n \leq 7$.

   (i) $g(x) = x^4 + x^3 + x^2 + x + 1$ (ii) $g(x) = x^3 + x^2 + 1$. *[(CO4)(Evaluate/HOCQ)]*

(6 + 6) = 12

Group - D

6. (a) Find the generator polynomial g(x) for a double error correcting binary BCH code of block length 15 over GF (16). Use primitive polynomial $p(x) = x^4+x+1$. *[(CO6)(Apply/IOCQ)]*

   (b) Find the Minimal Polynomial for the field element α^3 in GF (2^5). Use primitive polynomial $p(x) = x^5 + x^2 + 1$ to construct GF (2^5). *[(CO6)(Apply/IOCQ)]*

7 + 5 = 12

7. (a) Find out irreducible polynomials of degree 2 in GF(3). *[(CO6)(Apply/IOCQ)]*

   (b) Construct the addition and multiplication table on GF(4). *[(CO6)(Apply/IOCQ)]*

   (c) In GF(7), calculate $\frac{1}{4} - \frac{3}{5} + 2 - \frac{1}{6}$ *[(CO6)(Apply/IOCQ)]*

5 + 4 + 3 = 12

Group - E

8. A rate 1/3 convolutional coder with consistent length of '3' uses for generating vectors g1=(1 0 0), g2=(1 1 1) and g3=(1 0 1)

   (i) Sketch the encoder configuration.

   (ii) Draw the code tree (up to three levels), state diagram and Trellis diagram.

   (iii) If input message sequence is 10110, determine the output sequence of the encoder. *[(CO5)(Apply/IOCQ)]*

[2 + (4 + 2 + 2) + 2] = 12

9. For a convolutional code with the following generator polynomials: $g1(x)=1+x+x^2$ and $g2(x)=1+x$, determine the free distance of the code.
   (i) Define free distance in the context of convolutional codes.
   (ii) Calculate the code's free distance by listing possible error sequences and their corresponding weight.
   (iii) Verify your result by using the method of (weight enumerating the codewords.

[(CO5)(Apply/IOCQ)]

(6 + 3 + 3) = 12

Cognition Level	LOCQ	IOCQ	HOCQ
Percentage distribution	0	75	25