MATHEMATICS – I
(MTH1101)

Time Allotted : 2½ hrs **Full Marks : 60**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 4 (four) from Group B to E, taking one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A

1. Answer any twelve: **12 × 1 = 12**

Choose the correct alternative for the following

(i) Let $r(A)$ denote the rank of the matrix A. If all minors of a matrix A of order $r+1$ are zero, then
(a) $r(A) \leq r$ (b) $r(A) = 0$
(c) $r(A) > r$ (d) $r(A) = 1$.

(ii) The matrix $A = \begin{pmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \alpha \end{pmatrix}$ is orthogonal if $\alpha =$
(a) 1 (b) 0 (c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{2}$.

(iii) If the vector $(x+3y)\hat{\imath} + (y-2z)\hat{\jmath} + (x+mz)\hat{k}$ is solenoidal then the value of m is
(a) -1 (b) 1 (c) -2 (d) 2.

(iv) Identify the non-linear differential equation amongst the given equations :
(a) $\frac{d^2y}{dx^2} + 4y = 0$ (b) $y\frac{d^2y}{dx^2} - 4\frac{dy}{dx} = 0$
(c) $\frac{dy}{dx} - 3y = 0$ (d) $\frac{d^2y}{dx^2} + \tan x \frac{dy}{dx} - 2y = 0$

(v) Which of the following sequence is divergent?
(a) $\left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5} \ldots to\ \infty\right\}$. (b) $\left\{\left(1+\frac{1}{n}\right)^n\right\}, n \in N$
(c) $\left\{\frac{\sin n\pi}{n}\right\}, n \in N$ (d) $\{2^n\}, n \in N$.

(vi) If A and B are two non-zero square matrices such that $AB = O$, then
(a) A and B are non-singular (b) A is singular
(c) B is singular (d) A and B are singular.

(vii) $sin^{-1}\left(\frac{y}{x}\right)$ is a homogeneous function of degree

(a) 0 (b) 1 (c) 2 (d) 3.

(viii) $\lim_{y\to 0}\lim_{x\to 0}\frac{x+y}{x-y} =$

(a) 0 (b) -1 (c) 1 (d) 2.

(ix) $\frac{x\,dy - y\,dx}{x^2+y^2} =$

(a) $d\left(\frac{y}{x}\right)$ (b) $d\left\{log\left(\frac{y}{x}\right)\right\}$

(c) $d\left\{tan^{-1}\left(\frac{y}{x}\right)\right\}$ (d) $d\left\{tan^{-1}\left(\frac{x}{y}\right)\right\}$.

(x) If $f(u) = \log u + u^2$ and $u = u(x,y)$ then f_x =?

(a) $\frac{\partial f}{\partial x}$ (b) $\frac{\partial f}{\partial x}.u_x$ (c) $\frac{\partial f}{\partial u}$ (d) $\frac{\partial f}{\partial u}.u_x$.

Fill in the blanks with the correct word

(xi) The value of x for which the matrix $\begin{pmatrix} x+2 & 3 \\ 2 & x+3 \end{pmatrix}$ has rank 1 is ____________.

(xii) The angle between the tangents to the curve $\vec{r} = t\hat{i} + 2t\hat{j} - t^2\hat{k}$ at the point $t = \pm 1$ is ___________.

(xiii) $\frac{1}{D^2+1}\sin 2x$ equals to ____________.

(xiv) If $x = r\,cos\theta, y = r\,sin\theta$ then $\frac{\partial(x,y)}{\partial(r,\theta)}$ is equal to ___________.

(xv) The series $1 + r + r^2 + r^3$ ... . is convergent if ____________.

Group - B

2. (a) Determine the values of λ and μ for which the system of linear equations

$x + 2y + 3z = 4$

$x + 3y + 4z = 5$

$x + 3y + \lambda z = \mu$

have (i) no solution, (ii) a unique solution and (iii) an infinite number of solutions.

[(MTH1101.1, MTH1101.2)(Analyse /IOCQ)]

(b) If A , B are two n-th order square matrices and B is non-singular, prove that A and $B^{-1}AB$ have the same eigenvalues. *[(MTH1101.1,MTH1101.2)(Evaluate/HOCQ)]*

6 + 6 = 12

3. (a) If $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$, then show that $A^{n+2} - A^n - A^2 + I = O$, by Cayley-Hamilton theorem. Hence find A^{100}. *[(MTH1101.1, MTH1101.2)(Evaluate /HOCQ)]*

(b) Find the eigenvalues and eigenvectors of the matrix $A = \begin{bmatrix} 2 & 4 \\ 3 & 1 \end{bmatrix}$.

[(MTH1101.1, MTH1101.2)(Understand /LOCQ)]

(6 + 2) + 4 = 12

Group - C

4. (a) Consider a sequence $\{u_n\}, n \in N$ defined as follows:
$u_n = \frac{1}{n^2} + \frac{1}{(n+1)^2} + \frac{1}{(n+3)^2} + \cdots \frac{1}{(2n)^2}$.
Show that $\{u_n\}$ is monotonically decreasing and bounded.
[(MTH1101.3, MTH1101.4)(Understand/LOCQ)]

(b) Show that $\vec{A} = (6xy + z^3)\hat{\imath} + (3x^2 - z)\hat{\jmath} + (3xz^2 - y)\hat{k}$ is irrotational. Find a scalar function φ such that $\vec{A} = \vec{\nabla}\varphi$. *[(MTH1101.3, MTH1101.4)(Apply/IOCQ)]*

6 + 6 = 12

5. (a) Find the directional derivative of $\varphi = 5x^2y - 5y^2z + 2z^2x$ at the point $P(1,1,1)$ in the direction of the line $\frac{x-1}{2} = \frac{y-1}{-2} = z$. *[(MTH1101.3, MTH1101.4)(Evaluate/HOCQ)]*

(b) Test the convergence of the series $\frac{x}{y} + \frac{x(x+1)}{y(y+1)} + \frac{x(x+1)(x+2)}{y(y+1)(y+2)} + \cdots$ *to* ∞, $x > 0, y > 0$. *[(MTH1101.3, MTH1101.4)(Analyse/IOCQ)]*

6 + 6 = 12

Group - D

6. (a) Solve the following ordinary differential equation: $p - \frac{1}{p} = \frac{x}{y} - \frac{y}{x}$ where $p = \frac{dy}{dx}$.
[(MTH1101.5)(Understand/LOCQ)]

(b) Solve the differential equation $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = x^2e^{3x}$ using D-operator method. *[(MTH1101.5)(Apply/IOCQ)]*

6 + 6 = 12

7. (a) Solve the following Cauchy-Euler equation: $x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} - 3y = x^2 logx$.
[(MTH1101.5)(Remember/LOCQ)]

(b) Find the general and the singular solution of the equation $p = log\,(px - y)$ where , $p = \frac{dy}{dx}$. *[(MTH1101.5) (Apply/IOCQ)]*

6 + 6 = 12

Group - E

8. (a) Consider the function $f(x,y) = \frac{x^2y^2}{x^2y^2 + (x^2 - y^2)^2}$. Show that the double limit of $f(x,y)$ does not exist as $(x,y) \to (0,0)$. *[(MTH1101.6)(Analyse/IOCQ)]*

(b) Show that $f(x,y) = \begin{cases} \frac{2xy}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$ is not continuous at $(0,0)$.
[(MTH1101.6)(Analyse/IOCQ)]

(c) Use Divergence theorem to evaluate $\iint_S \{xz^2dy\,dz + (x^2y - z^3)dz\,dx + (2xy + y^3z)dx\,dy\}$ where S is the surface of the hemispherical region bounded by $z = \sqrt{a^2 - x^2 - y^2}$ and $z = 0$. *[(MTH1101.6)(Evaluate/HOCQ)]*

3 + 3 + 6 = 12

9. (a) If $u = xf\left(\frac{y}{x}\right) + g\left(\frac{y}{x}\right)$, then show that

(i) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = xf\left(\frac{y}{x}\right)$ (ii) $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = 0.$

[(MTH1101.6)(Remember/LOCQ)]

(b) Verify Green's theorem for $\oint_C \{(3x - 8y^2)dx + (4y - 6\text{xy})dy\}$ where C is the boundary of the region bounded by $x = 0$ and $y = 0$ and $x + y = 1$.

[(MTH1101.6)(Apply/IOCQ)]

6 + 6 = 12

Cognition Level	LOCQ	IOCQ	HOCQ
Percentage distribution	29.17	43.75	27.08