DISCRETE MATHEMATICS
(MTH2103)

Time Allotted : 2½ hrs **Full Marks : 60**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 4 (four) from Group B to E, taking one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A

1. Answer any twelve: **12 × 1 = 12**

Choose the correct alternative for the following

(i) A loop in a graph G gives one of the following in its dual
(a) pendant edge (b) loop
(c) an isolated vertex (d) two parallel edges

(ii) The chromatic number of a graph containing an odd circuit is
(a) 3 (b) 2
(c) greater than or equal to 3 (d) less than or equal to 3

(iii) The number of regions determined by the graph  is
(a) 1 (b) 2 (c) 3 (d) 0

(iv) If $4 \equiv 10 (\text{mod}\, n)$ for some positive integer n, then which of the following congruence may not always be true?
(a) $64 \equiv 1000 (\text{mod}\, n)$ (b) $28 \equiv 70 (\text{mod}\, n)$
(c) $-2 \equiv 4 (\text{mod}\, n)$ (d) $2 \equiv 5 (\text{mod}\, n)$

(v) If $\gcd(a, b) = 1$, then gcd (a^2, b^2) is
(a) 1 (b) 2 (c) 3 (d) 4

(vi) The remainder when 3^{10} is divided by 7 is
(a) 0 (b) 1 (c) 7 (d) 4

(vii) Total number of non-negative integral solutions to the equation $x_1 + x_2 = 10,\ x_1, x_2 \geq 0$ is
(a) 10 (b) 11 (c) 9 (d) 12

(viii) The number of permutations of the letters of BAT and $BALL$ are
(a) $3!\ and\ 4!$ (b) $3!\ and\ 3!$
(c) $3!\ and\ 12$ (d) $10\ and\ 20$

(ix) Converse of $'\sim p \rightarrow q'$ is
(a) $p \rightarrow \sim q$ (b) $q \rightarrow p$ (c) $\sim q \rightarrow \sim p$ (d) $q \rightarrow \sim p$

(x) Let p be a proposition 'He is intelligent' and q be a proposition 'He is tall'. Then $\sim q \wedge \sim p$ states that

(a) He is either intelligent or tall (b) He is neither tall nor intelligent

(c) He is not intelligent (d) He is intelligent and tall

Fill in the blanks with the correct word

(xi) A single vertex graph is ______________ chromatic.

(xii) The negation of $\sim(\sim p) \equiv$ ________________.

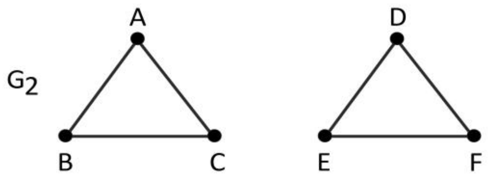
(xiii) The greatest common divisor of 123456 and 123457 is ____________ .

(xiv) The generating function for the sequence $1, 3, 3^3, 3^3, \ldots$ (given $|3x| < 1$) is ______.

(xv) The number of ways in which a team of 11 players can be selected from 22 players including 2 of them and excluding 4 of them is _____________ .

Group - B

2. (a) Draw the dual of the following disconnected graph:



[(MTH2103.1, MTH2103.2)(Apply/IOCQ)]

(b) Show that a connected self-dual planar graph with n vertices has $2n - 2$ edges.

[(MTH2103.1, MATH2103.2)(Create/HOCQ)]

(c) Show that the number of pendant vertices in a binary tree is $\frac{n+1}{2}$, where n is the no. of vertices in the tree. *[(MTH2103.1, MTH2103.2)(Remember/LOCQ)]*

4 + 4 + 4 = 12

3. (a) Using Decomposition theorem, find the chromatic polynomial and hence the chromatic number for the graph:

[(MTH2103.1, MTH2103.2)(Analyse/IOCQ)]

(b) How many ways a tree with 5 vertices can be coloured with at most 4 colours?

[(MTH2103.1, MTH2103.2)(Remember/LOCQ)]

(c) Is $\lambda^5 - 4\lambda^4 + 3\lambda^3 + 4\lambda$ a chromatic polynomial? Justify your answer.

[(MTH2103.1, MTH2103.2)(Apply/IOCQ)]

7 + 3 + 2 = 12

Group - C

4. (a) Prove that $(a+1)^3 - a^3$ is odd, for every integer a. *[(MTH2103.3)(Analyse/IOCQ)]*

(b) Prove that, if a is odd, then $24 | a(a^2 - 1)$. *[(MTH2103.3)(Evaluate/HOCQ)]*

6 + 6 = 12

5. (a) Use the Euclidean algorithm to find the greatest common divisor of 44 & 17. Express it in the form $44x + 17y$, where x & y are integers. Show your work. *[(MTH2103.3)(Evaluate/HOCQ)]*

(b) Find the remainder when 4444^{4444} is divided by 9. *[(MTH2103.3)(Analyse/IOCQ)]*

6 + 6 = 12

Group - D

6. (a) Five gentlemen A, B, C, D and E attend a party, where before joining the party, they leave their overcoats in a cloak room. After the party, the overcoats get mixed up and are returned to the gentlemen in a random manner. Using the principle of inclusion-exclusion, find the probability that none receives his own overcoat. *[(MTH2103.4)(Analyse/IOCQ)]*

(b) Find the number of integers between 1 and 250 both inclusive that are not divisible by any of the integers 2, 3, 5, and 7. *[(MTH2103.4)(Apply/IOCQ)]*

6 + 6 = 12

7. (a) State the Pigeonhole Principle. Use it to prove that if m is an odd positive integer, then there exists a positive integer n such that m divides $2^n - 1$. *[(MTH2103.4)(Create/HOCQ)]*

(b) Use the method of generating function to solve the following recurrence relation: $a_n = 4a_{n-1} + 3n.\ 2^n;\ n \geq 1,$ given that $a_0 = 4$. *[(MTH2103.4)(Apply/IOCQ)]*

6 + 6 = 12

Group - E

8. (a) Prove the following equivalence without using truth table: $p \leftrightarrow q \equiv (p \vee q) \rightarrow (p \wedge q)$. *[(MTH2103.5, MTH2103.6)(Analyse/IOCQ)]*

(b) Show that the following proposition is a contradiction by using truth table: $(\sim p \wedge q) \wedge (\sim q \wedge (p \rightarrow q))$. *[(MTH2103.5, MTH2103.6)(Evaluate/HOCQ)]*

6 + 6 = 12

9. (a) Without using truth table, find the principal disjunctive normal form of the following statement: $p \vee \sim q$. *[(MTH2103.5, MTH2103.6)(Analyse/IOCQ)]*

(b) Without using truth table, find the principal conjunctive normal form of the following statement: $(p \wedge q) \vee (\sim p \wedge q \wedge r)$. *[(MTH2103.5, MTH2103.6)(Evaluate/HOCQ)]*

6 + 6 = 12

Cognition Level	LOCQ	IOCQ	HOCQ
Percentage distribution	7.29	57.29	35.42