(vii) The linear equation 51x+6y=8 has no integral solution because gcd(51,6)=3 and
(a) 3 does not divide 8 (b) 3 divides 51
(c) 3 divides 6 (d) 51 does not divide 8.

(viii) The index of a subgroup is 5 and its order is 3. The order of the group is
(a) 8 (b) 10 (c) 15 (d) none of these.

(ix) A divisor of zero in Z_{10}, the ring of integers modulo 10, is
(a) [5] (b) [3] (c) [7] (d) [9].

(x) The only generator(s) of the cyclic group (Z, +) is / are
(a) 1 (b) 0,1 (c) 1, –1 (d) none of these.

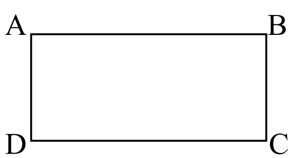
Group - B

2. (a) (i) Let G be a graph. Prove that the constant term in its chromatic polynomial is 0.
(ii) Let G be a graph which has more than one edge. Prove that the sum of the coefficients in its chromatic polynomial is 0.

(b) State Euler's Formula for simple connected planar graphs. Let G be a simple connected planar graph having n vertices, e edges and f regions (faces). Then prove that $e \geq \frac{3}{2}f$.

(3+3) + 6 = 12

3. (a) State the Decomposition Theorem. Use it to find the chromatic polynomial of the following graph. Show your work in detail.



(b) (i) State Hall's Marriage Theorem
(ii) Write down all the perfect matchings in K_4, the complete graph having 4 vertices (Name the vertices as A, B, C, D).

7+ (2+3) = 12

Group - C

4. (a) Consider the operation $a*b=a^b$, $\forall a,b \in N$. Examine whether the given operation is a binary operation on $\in N$. Is the given operation associative?
(b) Consider the binary operation a*b=a, $\forall a,b \in N$. Find the identity element in N under *, if it exists.
(c) Prove that the identity element in a group is unique.
(d) Show that the set of all odd integers does not form a group under usual addition.

4 + 3 + 3 + 2 = 12

5. (a) (i) Describe the set of all permutations on the set $\{1,2,3\}$. Which of them are even?
(ii) If G is a group such that $a^2 = e$ for all $a \in G$. Show that G is abelian. Is it true if $a^3 = e$, for all $a \in G$?

(b) Let G be a group with a finite number of elements. Show that for any $a \in b$, there exists an $n \in \mathbb{Z}^+$ such that $a^n = e$.

(3+5) + 4 = 12

Group - D

6. (a) Prove that the necessary and sufficient condition for a nonempty subset H of a group G to be a subgroup is that for all $a,b \in G$, $ab^{-1} \in G$.

(b) State and prove Lagrange's Theorem regarding the order of a subgroup of a finite group.

6 + 6 = 12

7. (a) (i) Prove that $(\mathbb{Q}, +)$ is a non-cyclic group.
(ii) Prove that every non-trivial subgroup of an infinite cyclic group is infinite.

(b) Prove that the centre of a group G, $Z(G) = \{x \in G : xg = gx \text{ for all } g \in G\}$ is a subgroup of G.

(3+5) + 4= 12

Group - E

8. (a) (i) Is Z_8={0,1,2,3,4,5,6,7} an integral domain? Give reasons for your answer.
(ii) Is Z_5= {0,1,2,3,4} a field? Give reasons for your answer.

(b) Prove that the intersections of two subrings is a subring.

(3+3) + 6 = 12

9. (a) Let K be a ring. The centre of K is the set $\{x \in K \mid ax = xa \; for \; all \; a \in K\}$. Prove that the centre of K is a subring of it.

(b) (i) Let K be a ring. Prove that $a^2 - b^2 = (a+b)(a-b)$ for all a, b in K if and only if K is commutative.

(ii) Suppose that there is a positive even integer n such that $a^n = a$ for all elements a of some ring K. Prove that $-a = a$ for all a in K.

5+ (3+4) = 12

2016

GRAPH THEORY AND ALGEBRAIC STRUCTURES
(MATH 2203)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternatives for the following: **10 × 1=10**

(i) Which of the following operations is not commutative?
(a) Matrix addition (b) arithmetical multiplication
(c) matrix multiplications (d) arithmetical addition.

(ii) If G has three vertices and no edges then the chromatic number of G is
(a) 2 (b) 0 (c) 3 (d) none of these.

(iii) Let G be a group and a∈G. If O(a)=17 then O(a^8) is
(a) 17 (b) 16 (c) 8 (d) 5.

(iv) Which of the following is not a subring of the ring of all integers under + and ×?
(a) The set of all even integers
(b) The set of all integers which are multiples of 3
(c) The set of all odd integers
(d) The set of integers which are multiples of 4.

(v) The symmetric group S_3 is
(a) cyclic but not abelian (b) cyclic and abelian
(c) non cyclic and non abelian (d) none of these.

(vi) The number of subrings of Z, the ring of all integers, is
(a) 2 (b) 3 (c) 4 (d) infinite.