OPERATIONS RESEARCH
(MTH2203)

Time Allotted : 2½ hrs **Full Marks : 60**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 4 (four) from Group B to E, taking one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A

1. Answer any twelve: **12 × 1 = 12**

Choose the correct alternative for the following

(i) If the primal has a feasible solution and the dual problem does not have feasible solution, then
(a) dual objective function is unbounded
(b) finite optimal for both exists
(c) primal objective function is unbounded
(d) finite optimal for both do not exist.

(ii) For a maximization model in an LPP, the coefficient of an artificial variable is
(a) 1 (b) 0 (c) M (d) $-M$

(iii) The range of λ for which the following payoff matrix is strictly determinable is

PLAYER B

λ	6	2	
-1	λ	-7	**PLAYER A**
-2	4	λ	

(a) $\lambda \geq -1$ (b) $\lambda \leq 2$ (c) $-1 \leq \lambda \leq 2$ (d) for any value of λ.

(iv) One disadvantage of using North-west corner rule to find IBFS is
(a) it is complicated to use
(b) it does not take into account the cost of transportation
(c) it leads to degenerate initial solution
(d) the solution is always infeasible.

(v) Which of the following statements is TRUE?
(a) In a two person zero sum game gains of one player are equal to the losses of other player.
(b) Every matrix game has a saddle point.
(c) Use the concept of dominance in reducing the size of a matrix game may lead to the loss of the saddle point.
(d) If a game has a saddle point, then the players play with their mixed

(vi) Which of the following Hessian matrices is positive definite?

(a) $\begin{pmatrix} 2 & 0 \\ 0 & -2 \end{pmatrix}$ (b) $\begin{pmatrix} 0 & 2 \\ 1 & 0 \end{pmatrix}$

(c) $\begin{pmatrix} 2 & -2 \\ -1 & 3 \end{pmatrix}$ (d) $\begin{pmatrix} -2 & 0 \\ 0 & -1 \end{pmatrix}$.

(vii) If a function is concave on a convex domain then any local maximum is

(a) global maximum (b) global minimum

(c) local maximum (d) neither global maximum nor global minimum

(viii) Let $Q(x, y, z)$ be a quadratic form such that $Q(x, y, z) \geq 0$ for $(x, y, z) \neq 0$ and $Q(-1, -2, 3) = 0$, then

(a) $Q(x, y, z)$could be indefinite

(b) $Q(x, y, z)$ could be positive definite

(c) $Q(x, y, z)$ could be negative semi definite

(d) $Q(x, y, z)$could be positive semi definite.

(ix) The reduction ratio in Dichotomous Search algorithm, if we go through r iterations is

(a) $2^{r/2}$ (b) $2^{r/3}$ (c) 2^{r^2} (d) 2^r

(x) Out of the following search algorithms, which one has the slowest rate of convergence:

(a) Interval halving method (b) Dichotomous search

(c) Fibonacci search (d) Golden section search

Fill in the blanks with the correct word

(xi) A basic solution that satisfies the non-negativity condition is known as ____________________ solution.

(xii) While solving an L.P.P using graphical method, the area bounded by the constraints in the positive quadrant is called the ______________________________.

(xiii) The function $f(x) = 2x^3 - 3x^2$ is convex, for all ________. (write the range of x)

(xiv) The nature of the quadratic form $Q(x, y, z) = x^2 + y^2 + z^2 + 3yz$ is ____________.

(xv) In Fibonacci Search algorithm, if 6 iterations are performed, the reduction ratio is given by __________.

Group - B

2. (a) Find the graphical solution of the given L.P.P :

Maximize $z = 2x_1 + x_2$

subject to the constraints

$x_1 + 2x_2 \leq 10$

$x_1 + x_2 \leq 6$

$x_1 - x_2 \leq 2$

$x_1 - 2x_2 \leq 1$

$x_1, x_2 \geq 0.$ *[(MTH2203.1, MTH2203.2)(Understand/LOCQ)]*

(b) Solve the following L.P.P by simplex algorithm:

Maximize $z = 2x_1 + 3x_2$

subject to the constraints
$x_1 + x_2 \leq 1$
$3x_1 + x_2 \leq 4$
$x_1, x_2 \geq 0.$

[(MTH2203.1, MTH2203.2)(Apply/IOCQ)]

5 + 7 = 12

3. (a) Solve the following L.P.P by Big-M method:
Maximize $z = 2x_1 + 4x_2$
subject to the constraints
$x_1 + 2x_2 \leq 2$
$2x_1 + 6x_2 \geq 12$
$x_1,\ x_2 \geq 0$

[(MTH2203.1, MTH2203.2)(Apply/IOCQ)]

(b) Find the dual of the given LPP:
Maximize $z = 10x_1 + 20x_2$
subject to the constraints
$x_1 + x_2 = 4$
$2x_1 - 3x_2 \leq 7$
$x_1,\ x_2 \geq 0.$

[(MTH2203.1, MTH2203.2)(Remember/LOCQ)]

7 + 5 = 12

Group - C

4. (a) Solve the given transportation problem using VAM and hence find its optimal solution.

	I	II	III	Supply
A	5	1	7	10
B	6	4	6	80
C	3	2	5	15
D	5	3	2	40
Demand	75	20	50	

[(MTH2203.1, MTH2203.2, MTH2203.3, MTH2203.4) (Evaluate/HOCQ)]

(b) Solve the assignment problem where the cost of assigning jobs J_1 to J_4 to workers W_1 to W_4 is given below:

	J_1	J_2	J_3	J_4
W_1	10	15	24	30
W_2	16	20	28	10
W_3	12	18	30	16
W_4	9	24	32	18

[(MTH2203.1, MTH2203.2, MTH2203.3, MTH2203.4) (Evaluate/HOCQ)]

7 + 5 = 12

5. (a) Use graphical method in solving the following game and find the value of the game.

PLAYER B

PLAYER A	2	2	3	−2
	4	3	2	6

[(MTH2203.1, MTH2203.2 MTH2203.3, MTH2203.4) (Apply/IOCQ)]

(b) Players A and B play a game in which each three coins, a 2 Rs., 5 Rs. and a 10 Rs. Each selects a coin without the knowledge of the other's choice. If the sum of the coins is an odd amount, then A wins B's coin. But, if the sum is even, then B win's A's coin. Find the best strategy for each player and the value of the game by dominance principal. *[(MTH2203.1, MTH2203.2 MTH2203.3, MTH2203.4)(Analyze/IOCQ)]*

6 + 6 = 12

Group - D

6. Use the method of Lagrangian multipliers to solve the following non- linear programming problem. Does the solution maximize or minimize the objective function?
Optimize $Z = 2x_1^2 - 24x_1 + 2x_2^2 - 8x_2 + 2x_3^2 - 12x_3 + 200$
Subject to the constraint
$x_1 + x_2 + x_3 = 11$
$x_1, x_2, x_3 \geq 0.$ *[(MTH2203.6) (Evaluate/HOCQ)]*

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7. (a) Use Kuhn-Tucker conditions to solve the following non-linear programming problem:
Maximize $z = -x_1^2 - x_2^2 - x_3^2 + 4x_1 + 6x_2$
subject to the constraints
$x_1 + x_2 \leq 2$
$2x_1 + 3x_2 \leq 12$
$x_1, x_2 \geq 0$ *[(MTH2203.6) (Evaluate/HOCQ)]*

(b) Determine the maximum or minimum (if any) of the following function:
$f(x, y, z) = 3x^2 + 2y^2 + z^2 - 2xy - 2xz + 2yz - 6x - 4y - 2z$
[(MTH2203.6) (Understand/LOCQ)]

8 + 4 = 12

Group - E

8. Find the minimum of $f(x) = x(x - 1.5)$ in the interval [0.0, 1.0] using Interval halving method taking the tolerance to be less than 0.15 *[(MTH2203.1, MTH2203.5) (Apply/IOCQ)]*

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9. Use Dichotomous Search method to minimize $f(x) = x^4 - 14x^3 + 60x^2 - 70x$ over $[0, 2]$. The tolerance limit being 0.3. Consider $\varepsilon = 0.001$. *[(MATH2203.1, MATH2203.5)(Apply/IOCQ)]*

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Cognition Level	LOCQ	IOCQ	HOCQ
Percentage distribution	14.58	52.09	33.33