

**STATISTICS FOR BUSINESS SYSTEM
(MTH2205)**

Time Allotted : 2½ hrs

Full Marks : 60

Figures out of the right margin indicate full marks.

*Candidates are required to answer Group A and
any 4 (four) from Group B to E, taking one from each group.*

Candidates are required to give answer in their own words as far as practicable.

Group - A

1. Answer any twelve:

12 × 1 = 12

Choose the correct alternative for the following

- (i) The moment generating function ($M_X(t)$) of Exponential distribution with mean $\frac{1}{2}$ is
 (a) $\frac{2}{2-t}$. (b) $\frac{2}{2-it}$. (c) $\frac{1}{1-2t}$. (d) $\frac{1}{1+2t}$.
- (ii) If the joint p. d. f. is given by $f(x, y) = \begin{cases} cx(1-x), & 0 \leq x \leq y \leq 1 \\ 0, & \text{otherwise} \end{cases}$, then the value of c is
 (a) 1. (b) 2. (c) 3. (d) 4.
- (iii) If $u + 3x = 5$, $2y - v = 7$ and correlation coefficient of x and y is 0.12, then what is the correlation coefficient of u and v ?
 (a) 0.12. (b) -0.12. (c) 1. (d) 0.
- (iv) If $r_{xy} = 0.6$, $\sigma_x = 3$, $\sigma_y = 4$, $\bar{x} = 13$, $\bar{y} = 17$, then the regression line of "y on x" is
 (a) $40x - 18y = 214$. (b) $8x - 10y + 66 = 0$.
 (c) $18x - 40y = 214$. (d) $10x - 8y + 66 = 0$.
- (v) In a test of hypothesis Type I error is committed when
 (a) null hypothesis is rejected when it was really false.
 (b) null hypothesis is rejected when it was really true.
 (c) null hypothesis is accepted when it was really false.
 (d) null hypothesis is accepted when it was really true.
- (vi) For a 99% confidence interval, the confidence coefficient is
 (a) 0.001. (b) 0.099. (c) 0.01. (d) 0.99.
- (vii) For a χ^2 variate if $P(0 < \chi^2 < 3) = 0.64$, then $P(-1 < \chi^2 < 3) = ?$
 (a) 0.64. (b) 0.36. (c) 1. (d) 0.
- (viii) In one-way ANOVA, which of the following is used within the F-ratio as a measurement of the variance of individual observations?
 (a) SSTR (b) MSTR. (c) SSE. (d) MSE.
- (ix) Two samples of sizes 25 and 35 are independently drawn from two normal populations, where the unknown variances are assumed to be equal. The number of degrees of freedom for the equal-variances t-test statistic is:
 (a) 58. (b) 59. (c) 60. (d) 61.
- (x) If $x + 4y + 3 = 0$ and $4x + 9y + 5 = 0$ be the two regression lines, then the expectation of x is
 (a) -1. (b) 0. (c) 1. (d) 2.

Fill in the blanks with the correct word

- (xi) The moment generating function ($M_X(t)$) of Exponential distribution with mean $1/2$ is _____.
- (xii) A fair coin is tossed 30 times. Let the random variable X be the number of heads that appear. The best upper bound using Markov's inequality on the probability of getting 20 or more heads is _____.
- (xiii) If $b_{xy} = -\frac{1}{4}$ and $b_{yx} = -1$ then the correlation coefficient between X and Y (r_{xy}) is _____.
- (xiv) If a population has Poisson distribution with parameter m , then the maximum likelihood estimate of m is _____.
- (xv) The _____ of a statistical test is the probability of accepting H_A when H_0 is false.

Group - B

2. (a) Use Chebyshev's inequality to find how many times a fair coin must be tossed in order that the probability that the ratio of the number of heads to the number of tosses will lie between 0.45 and 0.55 will be at least 0.95. [(MTH2205.1)(Apply/IOCQ)]

- (b) The two-dimensional random variables X and Y have a joint probability density function
- $$f(x, y) = \begin{cases} 8xy, & 0 < x < y < 1 \\ 0, & \text{otherwise} \end{cases}$$
- (i) Find $P\left(X < \frac{1}{2}, Y < \frac{1}{4}\right)$.
- (ii) Find all the marginal and conditional probability density functions.
- [[MTH2205.1](Evaluate/HOCQ)]
- 6 + 6 = 12**

3. (a) Suppose that the moment generating function of the random variable X is $M_X(t) = \left(\frac{1+3e^t}{4}\right)^{10}$.
What is the mean and variance of X ? [[MTH2205.1](Understand/LOCQ)]
- (b) Following is the joint probability distribution of X and Y :

X \ Y	1	2	3
0	$\frac{1}{10}$	$\frac{3}{10}$	$\frac{1}{10}$
2	$\frac{1}{5}$	$\frac{1}{10}$	$\frac{1}{5}$

- (i) Is it a valid distribution? Give reason.
- (ii) Find the marginal probability mass function of X and Y .
- (iii) Are X and Y independent? Justify.
- [[MTH2205.1](Analyse/IOCQ)]
- 6 + 6 = 12**

Group - C

4. (a) The following table provides data about the percentage of students who have free university meal and their CGPA scores. Calculate the Spearman's Rank Correlation between the two and interpret the result.

State University	Pune	Chennai	Delhi	Kanpur	Goa	Indore	Guwahati
% of students having free meals	14.4	7.2	27.5	33.8	38.0	15.9	4.9
% of students scoring above 8.5 CGPA	54	64	44	32	37	68	62

- (b) Out of the two regression lines given by $x + 4y + 3 = 0$ and $4x + 9y + 5 = 0$, which one is the regression line of "y on x"? Justify your answer. Find the mean of x and mean of y . Find the correlation coefficient between x and y . Estimate also the value of x when $y = 1.3$. [[MTH2205.1, MTH2205.2, MTH2205.4](Analyse/IOCQ)]
- [[MTH2205.1, MTH2205.2, MTH2205.4](Evaluate/HOCQ)]
- 5 + 7 = 12**

5. (a) Find the correlation coefficient and obtain the equations of two lines of regression for the following data:

X	65	66	67	67	68	69	70	72
Y	67	68	65	68	72	72	69	71

- (b) Find the multiple linear regression equation of x_1 on x_2 and x_3 from the data relating to three variables given below:

x_1	4	6	7	9	13	15
x_2	15	12	8	6	4	3
x_3	20	24	20	14	10	4

[[MTH2205.1, MTH2205.2, MTH2205.4](Evaluate/HOCQ)]

7 + 5 = 12

Group - D

6. (a) Let p denotes the probability of getting a head when a given coin is tossed once. Suppose that the hypothesis $H_0: p = 0.5$ is rejected in favour of $H_1: p = 0.6$ if 10 trials result in 7 or more heads. Calculate the probabilities of Type-I and Type-II error. [[MTH2205.4, MTH2205.5](Analyse/IOCQ)]
- (b) A machine produces a component of a product with a standard deviation of 1.6 cm in length. A random sample of 64 components was selected from the output and this sample has a mean length of 90 cm. The customer will reject the part if it is either less than 88 cm or more than 92 cm. Does the 95% confidence interval for the true mean length of all the components produced ensure acceptance by the customer? [[MTH2205.4, MTH2205.5](Analyse/IOCQ)]
- 6 + 6 = 12**
7. (a) In order to test whether a coin is perfect, the coin is tossed 5 times. The null hypothesis of perfectness is rejected if more than 4 heads are obtained. What is the probability of Type I error? Find the probability of Type II error when the corresponding probability of head is 0.2. [[MTH2205.4, MTH2205.5](Analyse/IOCQ)]

- (b) The nicotine content of a particular brand of cigarette is 1.5 mg per cigarette as advertised. A smoker who uses this particular brand wants to verify if it is really 1.5 mg or more. With this purpose the nicotine contents of 35 cigarettes are measured. The sample mean based on 35 cigarettes is 1.61 mg per cigarette. The population standard deviation is 0.4 mg. Do the data serve enough evidence that the average nicotine content exceeds 1.5 mg? Test at 5% level of significance and determine a 95% confidence interval for the population mean and hence draw a conclusion.
- [[MTH2205.4, MTH2205.5](Analyse/IOCQ)]
- 6 + 6 = 12**

Group - E

8. (a) In a certain factory there are two different processes of manufacturing the same item. The average weight in a sample of 250 items produced from one process is found to be 120 grammes with a S.D. of 12 grammes; the corresponding figures in a sample of 400 items from the other process are 124 and 4. Compute the Standard Error of difference between the two sample means. Is this difference significant? Also find 99% confidence limits for the differenced in the average weights of items produced by the two processes.
- [[MTH2205.1, MTH2205.3, MTH2205.4, MTH2205.6](Analyse/HOCQ)]
- (b) Survey of 320 families with 5 children each revealed the following distribution :

No. of boys:	5	4	3	2	1	0
No. of girls:	0	1	2	3	4	5
No. of families:	14	56	110	88	40	12

Is this result consistent with the hypothesis that male and female births are equally probable?

[[MTH2205.1, MTH2205.3, MTH2205.4, MTH2205.6](Remember/LOCQ)]

6 + 6 = 12

9. (a) Pizza delivery times of two cities are given below:
City X: Number of delivery times observed $n = 28$, Variance $\sigma_X^2 = 38$
City Y: Number of delivery times observed $m = 25$, Variance $\sigma_Y^2 = 83$
Check if the delivery times of city X are lesser than city Y at a 0.05 α level.
- [[MTH2205.1, MTH2205.3, MTH2205.4, MTH2205.6](Analyse/IOCQ)]
- (b) To study the performance of three detergents and three different water temperatures, the following ‘whiteness’ reading were obtained with specially designed equipment:

Water Temperature	Detergent A	Detergent B	Detergent C
Cold water	57	55	67
Warm water	49	52	68
Hot water	54	46	48

Perform a two-way analysis of variance, using 5 percent level of significance.

[[MTH2205.1, MTH2205.3, MTH2205.4, MTH2205.6](Evaluate/HOCQ)]

6 + 6 = 12

Cognition Level	LOCQ	IOCQ	HOCQ
Percentage distribution	12.5	54.17	33.33

