

DESIGN AND ANALYSIS OF ALGORITHMS
(CBS2201)

Time Allotted : 2½ hrs

Full Marks : 60

Figures out of the right margin indicate full marks.

*Candidates are required to answer Group A and
any 4 (four) from Group B to E, taking one from each group.*

Candidates are required to give answer in their own words as far as practicable.

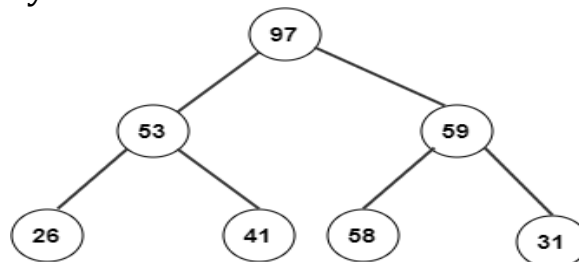
Group – A

1. Answer any twelve:

12 × 1 = 12

Choose the correct alternative for the following

- (i) What is the auxiliary space complexity of merge sort?
(a) $O(1)$ (b) $O(\log n)$ (c) $O(n)$ (d) $O(n \log n)$
- (ii) Apply Quick Sort on a given sequence 7 11 14 6 9 4 3 12. What is the sequence after first phase, pivot is first element?
(a) 6 4 3 7 11 9 14 12 (b) 6 3 4 7 9 14 11 12
(c) 7 6 14 11 9 4 3 12 (d) 7 6 4 3 9 14 11 12
- (iii) Consider the following heap after build-heap phase. What will be its corresponding array?



- (a) 26,53,41,97,58,59,31 (b) 26,31,41,53,58,59,97
(c) 26,41,53,97,31,58,59 (d) 97,53,59,26,41,58,31
- (iv) Which data structure is most suitable for implementing best first branch and bound strategy?
(a) stack (b) queue (c) priority queue (d) linked list
- (v) Which of the following is NOT true about a spanning tree of a graph?
(a) It must contain all vertices of the graph
(b) It must be connected
(c) It cannot contain cycles
(d) It must be a minimum-weight subgraph
- (vi) Best-First Search algorithm uses which of the following to make decisions?
(a) A stack (b) A queue
(c) A heuristic function (d) A cost function

- (vii) Which of the following problems can be solved using Dynamic Programming?
 (a) Longest Common Subsequence (b) Fractional Knapsack Problem
 (c) Activity Selection Problem (d) Depth First Search
- (viii) What is the time complexity of the Dynamic Programming solution to the Travelling Salesperson Problem (TSP)?
 (a) $O(n^2)$ (b) $O(n^3)$ (c) $O(2^n)$ (d) $O(n \log n)$
- (ix) The N-Queens problem is a classic example of a problem that can be solved using backtracking. In this problem, how many queens must be placed on an $N \times N$ chessboard such that no two queens attack each other?
 (a) N (b) N-1 (c) 2N-1 (d) 2N
- (x) Which technique is typically used in backtracking to solve combinatorial optimization problems?
 (a) Branch and bound (b) Greedy approach
 (c) Dynamic programming (d) Exhaustive search

Fill in the blanks with the correct word

- (xi) Greedy algorithms make _____ choices at each step that result in an optimal solution to the problem.
- (xii) The recurrence relation for the Fibonacci sequence $F(n) = F(n-1) + F(n-2)$ with base cases $F(0)=0$ and $F(1)=1$. The time complexity of the naïve recursive Fibonacci algorithm is _____.
- (xiii) The _____ algorithm is a dynamic programming approach that solves the longest common subsequence problem by filling a 2D table with solutions to subproblems
- (xiv) The _____ method for solving the N-Queens problem uses recursion to place queens one by one on the board, backtracking when a solution is not feasible.
- (xv) Big-Omega (Ω) notation is used to describe the _____ growth rate of an algorithm.

Group - B

2. (a) Calculate order of time complexity from the given piece of pseudo code using Frequency Count Method.

```

A() {
  n = 2^(2^k);
  for(i=1; i ≤ n; i++) {
    j = 2;
    while(j ≤ n) {
      j = j^2;
      print("HIT");
    }
  }
}

```

- (b) Suppose $P(n) = a_0 + a_1.n + a_2.n^2 + \dots + a_m.n^m$, that is the degree of $P(n)$ is m . Prove that $P(n) = O(n^m)$. [[C01](Apply/IOCQ)]
- (c) Prove that the time complexity of the function $f(n) = n!$, can't be expressed using the Big Theta notation (θ). [[C02](Apply/IOCQ)]

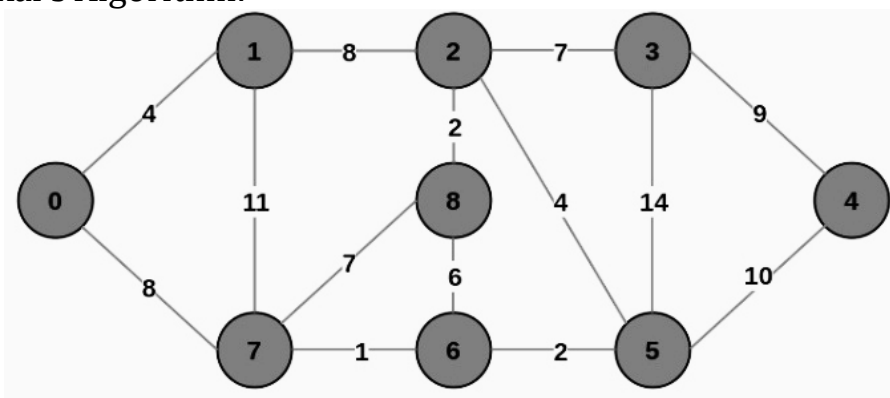
4 + 4 + 4 = 12

3. (a) State master's theorem and deduce the time complexity for the following recurrences using master's theorem:
 (i) $T(n) = 6T(n/4) + 2n$
 (ii) $T(n) = 8T(n/2) + n^2$. [[CO1](Apply/IOCQ)]
 (b) Find the generating function for the sequence 1, 4, 9, 16, Consider $a_0 = 1$. [[CO1](Apply/IOCQ)]
 (c) Explain stable and in-place sorting algorithm with example. [[CO2](Remember/LOCQ)]

(3 + 3) + 4 + 2 = 12

Group - C

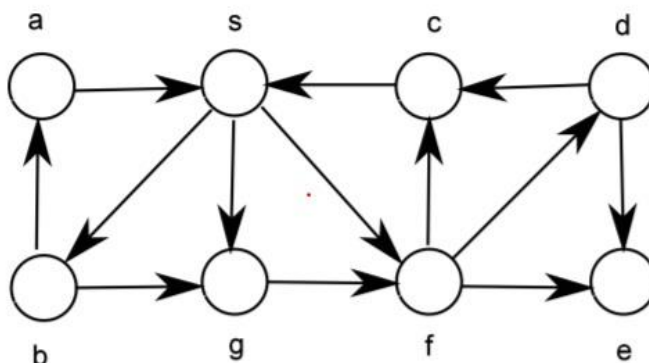
4. (a) Find out the optimal solution for the fractional Knapsack problem with capacity 60 is given below: Weight = {5, 10, 20, 30, 40} & Profit = {30, 20, 100, 90, 160} [[CO2](Apply/IOCQ)]
 (b) Find the Minimum Cost Spanning Tree (MST) of the given graph using Prim's and Kruskal's Algorithm:



- (c) Write down the Dijkstra's algorithm for finding out the shortest path. Prove that the time complexity of Dijkstra's algorithm for finding out the shortest path is $O(E \log V)$. [[CO2](Understand/LOCQ)]

3 + (3 + 3) + 3 = 12

5. (a) Let P be a shortest path from some vertex s to some other vertex t in a directed graph. If the weight of each edge in the graph is increased by one, will P still be a shortest path from s to t ? Justify your answer with proper explanation. [[CO3](CO6)(Analyse/IOCQ)]
 (b) What are the main advantages of using the Greedy Method for solving optimization problems? [[CO2](Remember/LOCQ)]
 (c) Traverse the following graph using BFS and DFS (show it's data structure updating in each step) where start node is 's'.

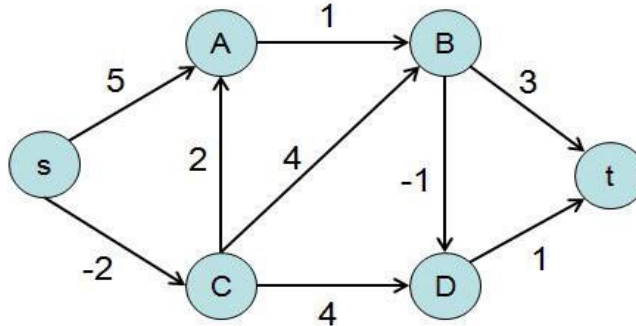


[[CO3](Apply/IOCQ)]

5 + 3 + 4 = 12

Group - D

6. (a) Define Dynamic Programming. How does it differ from Divide and Conquer? [[C03](Understand/LOCQ)]
 (b) State Max -Flow and Min-Cut theorem. [[C03](Remember/LOCQ)]
 (c) Consider the following graph and apply Bellman-Ford algorithm to find out the shortest-path from source vertex s.



[[C04](Apply/IOCQ)]
 $(2 + 2) + 2 + 6 = 12$

7. (a) Find out Minimum number of scalar multiplication required to multiply the following chain of matrices: A1 (5×15), A2 (15×10), A3 (10×5), A4 (5×25). [[C04](Apply/IOCQ)]
 (b) What is the Ford-Fulkerson algorithm for solving the Maximum Flow problem? Explain with an example. [[C04](Remember/LOCQ)]

$7 + 5 = 12$

Group - E

8. (a) Give a recursive algorithm MATRIX-CHAIN-MULTIPLY (A, s, i, j) that actually performs the optimal matrix-chain multiplication, given the sequence of matrices $\langle A_1, A_2, \dots, A_n \rangle$, the s table computed by MATRIX-CHAIN-ORDER, and the indices i and j. (The initial call would be MATRIX-CHAIN-MULTIPLY (A, s, 1, n)). [[C06](Evaluate/HOCQ)]
 (b) Discuss the 8-Queens problem. How can it be solved using backtracking? [[C04](Remember/LOCQ)]
 (c) Explain how pruning is used in backtracking algorithms to enhance efficiency. [[C03](Understand/LOCQ)]

$5 + 5 + 2 = 12$

9. (a) Write a backtracking algorithm to find all m- coloring of a graph with n vertices. [[C04](Understand/LOCQ)]
 (b) Briefly discuss the concepts of Approximation Ratio and Polynomial Time in case of Approximation Algorithms. [[C05](Understand/LOCQ)]
 (c) What is Polynomial Reduction? [[C05](Remember/LOCQ)]

$6 + 4 + 2 = 12$

Cognition Level	LOCQ	IOCQ	HOCQ
Percentage distribution	39.58	55.21	05.21