MATHEMATICAL METHODS
(MTH2001)

Time Allotted : 2½ hrs **Full Marks : 60**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 4 (four) from Group B to E, taking one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A

1. Answer any twelve: **12 × 1 = 12**

Choose the correct alternative for the following

(i) For an infinite power series $\sum_{n=0}^{\infty} c_n x^n$ if $\lim_{n\to\infty}\left|\frac{c_m}{c_{m+1}}\right| = \mathrm{R}$ then the radius of convergence is
(a) 1/R (b) R (c) 1 (d) ∞

(ii) The Frobenius method is particularly useful for solving differential equations when:
(a) The equation has constant coefficients.
(b) The equation has variable coefficients with a regular singular point.
(c) The equation has a regular point.
(d) The equation is non-linear.

(iii) For a differential equation with a regular singular point at x = 0, the indicial equation is obtained by:
(a) Substituting the power series into the differential equation and setting the lowest power of x to zero.
(b) Differentiating the power series twice.
(c) Integrating the power series.
(d) Finding the roots of the characteristic polynomial.

(iv) Which of the following recurrence relations do Legendre polynomials satisfy?
(a) $(n+1)P_{n+1}(x) - (2n+1)\mathrm{x}P_n(x) + \mathrm{n}P_{n-1}(x) = 0.$
(b) $(2\mathrm{n}+1)\mathrm{xP_n(x)} - \mathrm{nP_{n-1}(x)} = (\mathrm{n}+1)\mathrm{P_{n+1}(x)} = 0.$
(c) $(n+1)P_n(x) - P_{n+1}(x) + \mathrm{n}P_{n-1}(x) = 0.$
(d) $(n+1)P_{n+1}(x) - \mathrm{n}P_{n-1}(x) = 0.$

(v) What is the standard form of Bessel's differential equation?
(a) $x^2y'' + \mathrm{x}y' + (x^2 - n^2)\mathrm{y} = 0$ (b) $y'' + \mathrm{x}y' + (x^2 - n^2)\mathrm{y} = 0$
(c) $x^2y'' + 2\mathrm{x}y' + (x^2 - n)\mathrm{y} = 0$ (d) $y'' + y' + \mathrm{xy} = 0$

(vi) What is the differential equation satisfied by Hermite polynomials $H_n(x)$?
(a) $y'' - 2\mathrm{x}y' + 2\mathrm{ny} = 0$ (b) $y'' + 2\mathrm{x}y' - 2\mathrm{ny} = 0$
(c) $y'' - 2\mathrm{x}y' + 2\mathrm{y} = 0$ (d) $y'' + \mathrm{x}y' - \mathrm{ny} = 0$

(vii) Which of the following is true for a homogeneous PDE?
(a) It contains no derivatives.
(b) It has no boundary conditions.
(c) The PDE has all terms with the dependent variable and its derivatives.
(d) The PDE has the right-hand side equal to zero.

(viii) Lagrange's method is primarily used to solve:
(a) First-order linear partial differential equations.
(b) Second-order linear partial differential equations.
(c) Non-linear partial differential equations.
(d) Elliptic partial differential equations.

(ix) The heat equation in one dimension is given by:
(a) $u_t = c^2 u_{xx}$.
(b) $u_t + c^2 u_{xx} = 0$.
(c) $u_x = c^2 u_{tt}$.
(d) $u_x + c u_t = 0$.

(x) Complementary function of the given PDE $(D^2 - 7DD' + 6D'^2)z = 0$ is given by:
(a) $\phi_1(y + x) + \phi_2(y - 6x)$.
(b) $\phi_1(y + x) + \phi_2(y + 6x)$.
(c) $\phi_1(y + x).\phi_2(y + 6x)$.
(d) $\phi_1(y - x) + \phi_2(y - 6x)$.

Fill in the blanks with the correct word

(xi) The generating function for Hermite polynomials $H_n(x)$ is ______________.

(xii) For a non-integer n, the general solution of Bessel's equation is given by______________.

(xiii) In Lagrange's method, the characteristic equations are written as $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$, and their solutions are two independent ____________ of these equations.

(xiv) The system of ordinary differential equations in Charpit's method is obtained from the PDE by introducing the relations $\frac{dx}{ds} = \frac{\partial F}{\partial p}, \frac{dy}{ds} = \frac{\partial F}{\partial q}, \frac{dz}{ds} = p\frac{\partial F}{\partial p} + q\frac{\partial F}{\partial q}$, where s is the __________.

(xv) The Laplace transform is useful for solving boundary value problems because it converts partial differential equations into _____ differential equations.

Group - B

2. (a) Obtain the Frobenius series solution near $t = 0$ of

$$9t(1 - t)x'' - 12x' + 4x = 0$$

[(MTH2001.1, MTH2001.2)(Evaluate/HOCQ)]

(b) Show that infinity is not a regular singular point for the Bessel equation

$$x^2y'' + xy' + (x^2 - \alpha^2)y = 0$$

[(MTH2001.1,MTH2001.2)(Understanding/LOCQ)]

8 + 4 = 12

3. (a) Find the radius of convergence of the power series $\frac{x}{2} + \frac{1}{2}.\frac{3}{5}.x^2 + \frac{1}{2}.\frac{3}{5}.\frac{5}{8}x^3 + \cdots$

[(MTH2001.1, MTH2001.2) (Analyse/IOCQ)]

(b) Determine whether $x = 0$ is an ordinary point or a regular singular point of the differential equation $2x^2\frac{d^2y}{dx^2} + 7x(x + 1)\frac{dy}{dx} - 3y = 0$ (give proper explanation).

[(MTH2001.1, MTH2001.2) (Remember/LOCQ)]

(c) Find the indicial equation of $x^2y'' + (x + x^2)y' + (x - 9)y = 0$.
[(MTH2001.1, MTH2001.2) (Apply/IOCQ)]

4 + 4 + 4 = 12

Group - C

4. (a) Prove that $P_n(t) = \sum_{r=0}^{[\frac{n}{2}]} \frac{1}{2^n n!} \frac{n!}{r!} \frac{(-1)^r}{(n-r)!} \frac{(2n-2r)!}{(n-2r)!} t^{n-2r}$. *[(MTH2001.3)(Analyse/IOCQ)]*

(b) Show that $\sin(x) = 2J_1 - 2J_3 + 2J_5 - \cdots$ using the generating function of the Bessel functions. *[(MTH2001.3)(Application/IOCQ)]*

6 + 6 = 12

5. (a) Prove that $P_n'(1) = \frac{1}{2}n(n+1)$. *[(MTH2001.3)(Apply/IOCQ)]*

(b) Evaluate $\int_{-\infty}^{\infty} xe^{-x^2} H_n(x)H_m(x)\, dx$. *[(MTH2001.3)(Evaluate/HOCQ)]*

7 + 5 = 12

Group - D

6. (a) Solve the following partial differential equation by applying Lagrange's method: $(mz - ny)p + (nx - lz)q = ly - mx$, where l, m, n are constants and $p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}$. *[(MTH2001.4, MTH2001.5)(Understand/LOCQ)]*

(b) Construct a partial differential equation by eliminating the arbitrary function f from the relation: $z = e^{ax+by} f(ax + by)$. *[(MTH2001.4, MTH2001.5)(Apply/IOCQ)]*

6 + 6 = 12

7. (a) Obtain a partial differential equation by eliminating the arbitrary constants from the relation $z = a(x + y) + b(x - y) + abz + c$ where x and y are independent variables, and a, b, c are arbitrary constants. *[(MTH2001.4, MTH2001.5)(Understand/LOCQ)]*

(b) Find a partial differential equation by elimination of arbitrary function ϕ from $z = \phi\left(\frac{y}{x}\right)$. *[(MTH2001.4, MTH2001.5)(Remember/LOCQ)]*

(c) Solve $x^2p + y^2q = z^2$ where $p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}$. *[(MTH2001.4, MTH2001.5)(Analyse/IOCQ)]*

4 + 4 + 4 = 12

Group - E

8. (a) Solve $(D^2 - 2DD' + D'^2)z = e^{x+2y} + x^3$, where $D \equiv \frac{\partial}{\partial x}, D' \equiv \frac{\partial}{\partial y}$
[(MTH2001.5, MTH2001.6) (Remember/LOCQ)]

(b) Solve $(4D^2 - 4DD' + D'^2)z = \log(x + 2y)$, where $D \equiv \frac{\partial}{\partial x}, D' \equiv \frac{\partial}{\partial y}$
[(MTH2001.5, MTH2001.6) (Remember/LOCQ)]

6 + 6 = 12

9. (a) A tightly stretched string with fixed end points $x = 0$ and $x = l$ is pulled from its middle portion and the initial position is given by $y = y_0 \sin^3(\frac{\pi x}{l})$. It is released from rest at this position. Determine the displacement $y(x, t)$ of the string.
[(MTH2001.5, MTH2001.6) (Apply/IOCQ)]

(b) Find the particular integral of the equation $\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = \sin(2x + 3y)$.

[(MTH2001.5, MTH2001.6) (Remember/LOCQ)]

8 + 4 = 12

Cognition Level	LOCQ	IOCQ	HOCQ
Percentage distribution	39.58	46.58	13.54