

**MATHEMATICAL & STATISTICAL METHODS
(MTH2204)**

Time Allotted : 2½ hrs

Full Marks : 60

Figures out of the right margin indicate full marks.

***Candidates are required to answer Group A and
any 4 (four) from Group B to E, taking one from each group.***

Candidates are required to give answer in their own words as far as practicable.

Group – A

1. Answer any twelve:

12 × 1 = 12

Choose the correct alternative for the following

- (i) The method of characteristics is particularly useful for solving:
 - (a) Elliptic PDEs.
 - (b) Parabolic PDEs.
 - (c) First-order PDEs.
 - (d) Hyperbolic PDEs.
- (ii) Complementary function of the given PDE $(D^2 - 7DD' + 6D'^2)z = 0$ is given by:
 - (a) $\phi_1(y + x) + \phi_2(y - 6x)$.
 - (b) $\phi_1(y + x) + \phi_2(y + 6x)$.
 - (c) $\phi_1(y + x) \cdot \phi_2(y + 6x)$.
 - (d) $\phi_1(y - x) + \phi_2(y - 6x)$.
- (iii) Lagrange's Interpolation Formula is advantageous because:
 - (a) It does not require equally spaced data points.
 - (b) It requires only forward differences.
 - (c) It applies only to polynomial functions.
 - (d) It cannot be used for higher-order polynomials.
- (iv) Weddle's Rule is applicable when the number of subintervals is a multiple of:
 - (a) 2
 - (b) 3
 - (c) 6
 - (d) 4
- (v) The period of the function $f(x) = \cos(\sqrt{3}x)$ is
 - (a) 2π
 - (b) $\frac{\pi}{\sqrt{3}}$
 - (c) $\frac{2\pi}{3}$
 - (d) $\frac{2\pi}{\sqrt{3}}$
- (vi) The function $f(x) = \begin{cases} 1 + \frac{2x}{\pi}, & -\pi < x < 0 \\ 1 - \frac{2x}{\pi}, & 0 \leq x < \pi \end{cases}$ is symmetric about
 - (a) x -axis only.
 - (b) y -axis only.
 - (c) origin.
 - (d) both x and y -axes.
- (vii) In a Fourier series, the coefficients of the sine terms represent which part of the function?
 - (a) Even symmetry components.
 - (b) Odd symmetry components.
 - (c) Both even and odd components.
 - (d) None of the above.

- (viii) Which of the following distributions is used for modelling the number of rare events in a fixed interval of time or space?
 (a) Hypergeometric distribution. (b) Poisson distribution.
 (c) Uniform distribution. (d) Gamma distribution.
- (ix) For a perfectly symmetrical distribution, which of the following statements is true?
 (a) Mean = Median = Mode (b) Mean > Median > Mode
 (c) Mean < Median < Mode (d) Mode > Median > Mean
- (x) Spearman's Rank Correlation Coefficient is used when:
 (a) The data is normally distributed.
 (b) The relationship between variables is linear.
 (c) The data is ranked or ordinal.
 (d) The variables are independent.

Fill in the blanks with the correct word

- (xi) In the classification of second-order PDEs, the equation $u_{xx} + 2u_{yx} + u_{yy} = 0$ is _____.
- (xii) The degree of the interpolating polynomial for a function (x) whose values are known at 8 interpolating points is _____.
- (xiii) Simpson's 1/3 rule provides better accuracy than the Trapezoidal Rule by approximating the function with a _____ polynomial instead of straight-line segments.
- (xiv) The Fourier Series of an even function contains only _____ terms.
- (xv) Kurtosis describes the _____ of a distribution, indicating whether it has heavy or light tails compared to a normal distribution.

Group - B

2. (a) Find a complete integral of $px + qy = pq$ where $p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}$. [(MTH2204.5)(Evaluate/HOCQ)]
- (b) Eliminate the arbitrary constants a and b , and obtain the partial differential equation from the following relation: $z = ax^2 + by^2 + ab$. [(MTH2204.5)(Remember/LOCQ)]
- 7 + 5 = 12**
3. (a) Solve $(1 + y)p + (1 + x)q = z$ where $p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}$. [(MTH2204.5)(Apply/IOCQ)]
- (b) Find the differential equation of all spheres of radius c , having centres on xy -plane. [(MTH2204.5)(Create/HOCQ)]
- 6 + 6 = 12**

Group - C

4. (a) Establish the relation $\Delta + \nabla = \frac{\Delta}{\nabla} - \frac{\nabla}{\Delta}$ where Δ, ∇ represent forward and backward difference operators, respectively. [(MTH2204.1, MTH2204.2)(Understand/LOCQ)]

- (b) Estimate the population (in million) of a city in the year 1965 from the data given in the following table:

Year	1951	1961	1971	1981
Population (in million)	37	43	57	84

[(MTH2204.1, MTH2204.2)(Evaluate/HOCQ)]

6 + 6 = 12

5. (a) For the following data

x	-4	-2	0	2	4	6
f(x)	-139	-21	1	23	141	451

Construct the forward and backward difference tables. Using the corresponding interpolation, and show that the interpolating polynomials are same.

[(MTH2204.1, MTH2204.2)(Analyse/IOCQ)]

- (b) Find the value of $\int_0^1 e^x dx$ by Trapezoidal rule with $h = 0.1$. Hence, find the value of the absolute error in the solution.

[(MTH2204.1, MTH2204.2)(Evaluate/HOCQ)]

7 + 5 = 12

Group - D

6. (a) Using the Parseval's Identity for the Fourier series of $f(x) = \cos\left(\frac{x}{2}\right)$ in $(-\pi, \pi)$, prove that $\sum_{n=1}^{\infty} \frac{1}{(4n^2-1)^2} = \frac{\pi^2-8}{16}$.

[(MTH2204.4)(Evaluate/HOCQ)]

- (b) Find the Fourier *cosine* series of the function $f(x) = \begin{cases} x^2, & 0 \leq x < 2 \\ 4, & 2 \leq x \leq 4. \end{cases}$

[(MTH2204.4)(Remember/LOCQ)]

6 + 6 = 12

7. (a) Prove that the functions $\sin(nx)$ and $\cos(mx)$ are orthogonal over the interval $[-\pi, \pi]$.

[(MTH2204.4)(Analyse/IOCQ)]

- (b) Find the Fourier cosine series expansion of $f(x) = e^x$ in $0 \leq x \leq \pi$.

[(MTH2204.4)(Evaluate/HOCQ)]

6 + 6 = 12

Group - E

8. (a) Let X be a random variable following a continuous uniform distribution over the interval $[2, 8]$.

(i) Find the probability density function (PDF) of X .

(ii) Compute the probability that X takes a value between 3 and 6.

[(MTH2204.3, MTH2204.6)(Analyse/IOCQ)]

- (b) A group of 100 items has mean 60 and variance 25. If the mean of the first 50 items is 61 and the standard deviation is 4.5, find the mean and standard deviation of the other 50 items.

[(MTH2204.3, MTH2204.6)(Evaluate/HOCQ)]

5 + 7 = 12

9. (a) If a random variable X follows uniform distribution with parameters m and n , then find (i) mean of X (ii) $\text{Var}(X)$.

[(MTH2204.3, MTH2204.6)(Analyse/IOCQ)]

- (b) Out of two regression lines given by $x + 0.2y = 4.2$ and $y + 0.8x = 8.4$, which one is the regression line of " y on x "? Find the mean of x and y . Further find the correlation coefficient between x and y . [(MTH2204.3, MTH2204.6)(Evaluate/HOCQ)]

6 + 6 = 12

Cognition Level	LOCQ	IOCQ	HOCQ
Percentage distribution	17.71	31.25	51.04