

FORMAL LANGUAGE & AUTOMATA THEORY
(CSEN 3002)

Time Allotted : 2½ hrs

Full Marks : 60

Figures out of the right margin indicate full marks.

*Candidates are required to answer Group A and
any 4 (four) from Group B to E, taking one from each group.*

Candidates are required to give answer in their own words as far as practicable.

Group – A

1. Answer any twelve:

12 × 1 = 12

Choose the correct alternative for the following

- (i) If a nondeterministic finite automaton has N states then the corresponding equivalent deterministic finite automaton cannot have more than
(a) N states (b) $(N+1)$ states (c) $2 \times N$ states (d) 2^N states
- (ii) Which of the following is true about a DFA?
(a) It can have multiple initial states
(b) It can have multiple accepting states
(c) It allows ϵ -transitions
(d) It accepts context-free languages.
- (iii) What will be the minimum number of possible states of a DFA that accepts a language $L = \{w_1 0 w_2 \mid w_1, w_2 \in \{0, 1\}^*, |w_1|=2, |w_2| \geq 3\}$?
(a) 5 (b) 8 (c) 6 (d) 7
- (iv) Which of the following is true?
(a) $(01)^*0 = 0(10)^*$
(b) $(0+1)^*01(0+1)^*+1^*0^* = (0+1)^*$
(c) $(0+1)^*0(0+1)^*1(0+1) = (0+1)^*01(0+1)^*$
(d) All of the above
- (v) A regular language:
(a) is also a context free language (b) is also a linear language
(c) need not to be context free (d) both (a) and (b)
- (vi) Which of the following grammars generates a regular language?
(a) Right-linear grammar (b) Context-free grammar
(c) Context-sensitive grammar (d) Unrestricted grammar
- (vii) Which of the following is a CFL?
(a) $L_1 = \{a^n b^m c^k \mid m, n, k \geq 0 \text{ and } k = m + n\}$ (b) $L_2 = \{a^n b^{2n} \mid n \geq 0\}$
(c) $L_3 = \{a^n b^n c^{3n} \mid n \geq 0\}$ (d) Both (a) & (b)

- (viii) If a Context Free Language L_3 has a Type 2 grammar G which is ambiguous then there must be a terminal string α such that
- (a) α has two different derivation trees
 - (b) α has two different rightmost derivations
 - (c) α has two different leftmost derivations
 - (d) All of the above.
- (ix) Which machine model is used to define the concept of decidability?
- (a) Finite Automaton
 - (b) Pushdown Automaton
 - (c) Counter Machine
 - (d) Turing Machine
- (x) Recursive language is closed under which of the following operations such that Recursively Enumerable language is not closed under that operation?
- (a) Union
 - (b) Intersection
 - (c) Concatenation
 - (d) Complementation.

Fill in the blanks with the correct word

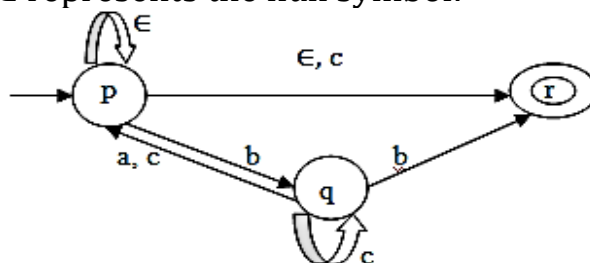
- (xi) The state transition function of a ϵ -NFA, $N_1 = (Q, q_0, \Sigma, F, \delta)$, is given by δ : _____.
- (xii) The complement of a regular language is a _____ language.
- (xiii) A pushdown automaton _____ be designed to accept the language $L = \{a^n b^n c^{2n} \mid n \geq 0\}$
- (xiv) A Turing Machine can be formally described by a 7-tuple, which includes a finite set of states, a finite set of input symbols, a _____, a transition function, a start state, an accept state, and a reject state.
- (xv) According to Arden's theorem, a regular expression $R = Q + RP$ has a unique solution given by _____.

Group - B

2. (a) Consider the following Mealy machine. Convert this machine to its equivalent Moore machine with the clear explanations of all the steps. [[CSEN3002.2](Apply/IOCQ)]

P.S.	Input = 0		Input = 1	
	N.S.	Output	N.S.	Output
$\rightarrow q_1$	q3	1	q2	1
q2	q1	0	q4	1
q3	q2	0	q1	0
q4	q4	0	q3	1

- (b) Define epsilon closure of a state in nondeterministic finite automata (NFA). Consider the following ϵ -NFA on $\Sigma = \{a, b, c\}$, where p is the initial state, r is the final state and ϵ represents the null symbol:



Find the equivalent DFA of the given ε – NFA.

[[CSEN3002.1](CSEN3002.2)(Remember/LOCQ)(Apply/IOCQ)]

6 + (1 + 5) = 12

3. (a) Define a finite state machine (FSM) and explain its components.
[[CSEN3002.1](Remember/LOCQ)]
- (b) Do you agree that the main limitation of FSMs is their finite memory? Justify your response.
[[CSEN3002.1](Analyse/IOCQ)]
- (c) Design a DFA that accepts strings over the alphabet $\{0, 1\}$ that ends with "110".
[[CSEN3002.6](Apply/IOCQ)]
- 4 + 4 + 4 = 12**

Group - C

4. (a) Consider the following language:
 $L = \{a^m b^n : m \geq 0 \text{ \& } n \geq 0 \text{ and } m=n\}$.
Is this language regular? If this language is regular then prove your conjecture by constructing a regular grammar/regular expression or deterministic finite automata which accepts this language. If this language is not regular then prove this hypothesis.
[[CSEN3002.5](Analyse/IOCQ)]
- (b) Derive a regular (Type 3) grammar for the regular expression $0^*1(0 + 10^*1)^*$
[[CSEN3002.6](Apply/IOCQ)]
- 6 + 6 = 12**
5. (a) Let L_1 be the following language defined on the input alphabet $\Sigma = \{0, 1\}$.
 $L_1 = \{\alpha \mid \text{the string } \alpha \text{ does not contain the substring '00'}\}$
Thus the strings 101101 and 01110 are both in L_1 , but the string 1001 is not in L_1 .
Now, answer the following questions:
(i) Give a regular expression for L_1 .
(ii) Design a NFA from the regular expression that you obtained at part (i).
[[CSEN3002.6](Apply/IOCQ)]
- (b) Consider the following language L_2 :
 $L_2 = \{1^{n^2} \mid n \geq 0\}$
Prove that L_2 is not a regular language by using pumping lemma.
[[CSEN3002.5](Evaluate/HOCQ)]
- (3 + 5) + 4 = 12**

Group - D

6. (a) Simplify the given CFG whose productions are as follows:
- $$\begin{aligned} S &\rightarrow ACD \\ A &\rightarrow a \mid F \\ B &\rightarrow \epsilon \\ C &\rightarrow ED \mid \epsilon \\ D &\rightarrow BC \mid b \\ E &\rightarrow b \\ F &\rightarrow aF \end{aligned}$$

[[CSEN3002.4] (Apply/IOCQ)]

- (b) Prove that if L_1 and L_2 are context free then $L_1 \cup L_2$ is also context free but $L_1 \cap L_2$ need not to be context free. [[CSEN3002.3](Analyze/IOCQ)]
6 + 6 = 12
7. (a) For each of the following given languages, mention which one is supposed to be accepted by a deterministic push down automata or non-deterministic push down automata or both or none of these.
- (i) $L(G) = \{ww^R \mid w \in \{a, b\}^*\}$
 - (ii) $L(G) = \{a^n b^{n+2} \mid n > 0\}$
 - (iii) $L(G) = \{a^n b^n c^m \mid n, m > 0\}$
 - (iv) $L(G) = \{a^{n+m} b^{n+m} c^m \mid n, m > 0\}$
 - (v) $L(G) = \{a^n b^m \mid n, m > 0\}$
- (b) A pushdown automaton (PDA) cannot accept the language $L = \{a^n b^n c^n \mid n \geq 1\}$. Justify with proper reasons. [[CSEN3002.3](Understand/LOCQ)]
[[CSEN3002.3](Analyze/HOCQ)]
(5 × 1) + 7 = 12

Group - E

8. (a) Design a Turing Machine M which accepts all odd length palindromes defined over the alphabet set $\{a, b\}$. [[CSEN3002.6](Apply/IOCQ)]
- (b) Design a Turing Machine M which accepts the language $L = \{w\#w \mid w \in \{a, b\}^*\}$ [[CSEN3002.6](Apply/IOCQ)]
6 + 6 = 12
9. (a) Design a Turing machine M that finds the 1's complement of a binary number. e.g. if the input number is 00110 then the output will be 11001. [[CSEN3002.6](Apply/IOCQ)]
- (b) Explain the differences between recursively enumerable and recursive language with suitable examples. [[CSEN3002.1](Understand/LOCQ)]
7 + 5 = 12

Cognition Level	LOCQ	IOCQ	HOCQ
Percentage distribution	11.46	77.08	11.46