COMPUTATIONAL MATHEMATICS
(MATH 3221)

Time Allotted : 2½ hrs **Full Marks : 60**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 4 (four) from Group B to E, taking one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A

1. Answer any twelve: **12 × 1 = 12**

Choose the correct alternative for the following

(i) The minimum number of moves that will transfer twenty disks from one peg to another in the Tower of Hanoi problem is
(a) 524287 (b) 2097151 (c) 4194303 (d) 1048575

(ii) $\lfloor -2\pi \rfloor$
(a) -6 (b) -7 (c) -3 (d) -5

(iii) $C(n,0)-C(n,1)+C(n,2)-\ldots+(-1)^n C(n,n)=$
(a) 1 (b) 0 (c) 2 (d) 2^n (Here n is a positive integer)

(iv) The Stirling number of the first kind $S_1(4,2)$ is
(a) 6 (b) 11 (c) 9 (d) 7

(v) $\varphi(32)=$
(a) 12 (b) 8 (c) 31 (d) 16

(vi) The remainder in the division of $96!$ by 97 is
(a) 94 (b) 1 (c) 95 (d) 96

(vii) The solution of the recurrence relations $a_n - 5a_{n-1} + 6a_{n-2} = 0$ is
(a) $a_n = C_1 5^n + C_2 6^n$ (b) $a_n = C_1 2^n + C_2 3^n$
(c) $a_n = C_1 2^n + C_2 5^n$ (d) $a_n = C_1 3^n + C_2 6^n$

(viii) The function $\frac{1}{1+x^2}$ is the generating function of the sequence
(a) $\{1,0,1,0,1,0,1,0,1,0\ldots\}$ (b) $\{1,-1,1,-1,1,-1\ldots\}$
(c) $\{2,0,2,0,2,0,2,0,2,0\ldots\}$ (d) $\{1,0,-1,0,1,0,-1,0,1,0\ldots\}$

(ix) If $S_n = \frac{1}{2}S_{n-1}$ and $1 < S_0 < 2$, then $\lim_{n\to\infty} S_n =$
(a) 1 (b) $\frac{1}{2}$ (c) $-\infty$ (d) 0

(x) Which one of the following is not a Fibonacci number?
(a) 55 (b) 233 (c) 144 (d) 376

Fill in the blanks with the correct word

(xi) The number of subsets of a set having 10 elements is ________.

(xii) $\Delta^6(x^{\overline{4}}) =$ ________.

(xiii) $C(100, 30) + C(100, 29) =$ ________.

(xiv) The exponential generating function of the Bernoulli numbers is ________.

(xv) The Eulerian number $E(4,\ 2)$ is ________.

Group - B

2. (a) Let T_n denote the minimum number of moves that will transfer n disks from one peg to another in the puzzle called the Tower of Hanoi. Prove that $T_n \geq 2T_{n-1} + 1$, where n is a positive integer. *[(MATH3221.1, MATH3221.5, MATH3221.6)(Understand/LOCQ)]*

(b) Recall that $\sum_{o \le k < n} k^{\underline{m}} = \left[\dfrac{k^{\underline{m+1}}}{m+1}\right]_0^n = \dfrac{n^{\underline{m+1}}}{m+1}$, where m, n are non-negative integers.

Use this result to prove that $\sum_{0 \le k < n} k^2 = \frac{1}{3} n(n - \frac{1}{2})(n-1)$.

[(MATH3221.1, MATH3221.5, MATH3221.6)(Analyse/IOCQ)]

7 + 5 = 12

3. (a) Let the operators Δ and ∇ be defined as follows:

$\Delta f(x) = f(x+1) - f(x)$,

$\nabla f(x) = f(x) - f(x-1)$.

Compute $\Delta^3 x^{\underline{m}}, \nabla^6 (x^{\overline{m}})$, where $m \geq 7$ an integer is.

[(MATH3221.1, MATH3221.5, MATH3221.6)(Analyse/IOCQ)]

(b) Evaluate the sums $S_n = \sum_{k=0}^{n} (-1)^{n-k}$ and $T_n = \sum_{k=0}^{n} (-1)^{n-k} k$ assuming that $n \geq 0$. *[(MATH3221.1, MATH3221.5, MATH3221.6)(Analyze/IOCQ)]*

6 + 6 = 12

Group - C

4. (a) Let $E(n, k)$ denote the Eulerian numbers. Prove that $E(n, k) = (k+1)E(n-1, k) + (n-k)E(n-1, k-1)$, where $n > 0$ is an integer.

[(MATH3221.2, MATH3221.5, MATH3221.6)(Apply/IOCQ)]

(b) Prove that $\gcd(km, kn) = k \gcd(m, n)$, where k, m, n are all positive integers.

[(MATH3221.2, MATH3221.5, MATH3221.6)(Apply/IOCQ)]

6 + 6 = 12

5. (a) Let F_k denote the Fibonacci numbers. Let $m \geq 2$ be a fixed integer. Prove that $F_{m+n} = F_{m-1}F_n + F_m F_{n+1}$ for all integers $n \geq 1$, by induction on n.

[(MATH3221.2, MATH3221.5, MATH3221.6)(Evaluate/HOCQ)]

(b) Let $r,\ k$ be positive integers and $r > k$. Prove that $(r-k)\binom{r}{k} = r\binom{r-1}{k}$.

[(MATH3221.2, MATH3221.5, MATH3221.6)(Apply/IOCQ)]

6 + 6 = 12

Group - D

6. (a) Prove that $lcm(km, kn) = k\ lcm(m, n)$, where k is a positive integer.

[(MATH3221.3, MATH3221.5, MATH3221.6)(Evaluate/HOCQ)]

(b) Prove, by mathematical induction, that if x is any real number other than 1, then $\sum_{j=0}^{n-1} x^j = 1 + x + x^2 + \cdots + x^{n-1} = \frac{x^n - 1}{x-1}$.

By using this result, prove that, if m and n are positive integers and if $m > 1$, then $n < m^n$.

[(MATH3221.3, MATH3221.5, MATH3221.6)(Evaluate/HOCQ)]

6 + 6 = 12

7. (a) Find the remainder in the division of $5^{801} + 6^{202} + 39!$ by 41. State every result that you use and show your calculations.

[(MATH3221.3, MATH3221.5, MATH3221.6)(Evaluate/HOCQ)]

(b) Prove that $\left\lfloor \sqrt{\lfloor x \rfloor} \right\rfloor = \left\lfloor \sqrt{x} \right\rfloor$, where x is a positive real number.

[(MATH3221.3, MATH3221.5, MATH3221.6)(Remember/LOCQ)]

6 + 6 = 12

Group - E

8. (a) A particle is moving in the horizontal direction. The distance it travels in each second is equal to two times the distance it travelled in the previous second. If a_r denotes the position of the particle in the rth second, determine a_r, given that $a_0 = 3$ and $a_3 = 10$.

[(MATH3221.4, MATH3221.5, MATH3221.6) (Create/HOCQ)]

(b) Find the coefficient of X^{20} in $(X^3 + X^4 + X^5 + \cdots)^5$.

[(MATH3221.4, MATH3221.5, MATH3221.6) (Analyse/IOCQ)]

6 + 6 = 12

9. (a) Use the method of generating function to solve the recurrence relation $a_n = 4a_{n-1} - 4a_{n-2} + 4^n;\ n \geq 2,$ given that $a_0 = 2, a_1 = 8.$

[(MATH3221.4, MATH3221.5, MATH3221.6)(Evaluate/HOCQ)]

(b) Solve the recurrence relation

$$a_n = 5\,a_{n-1} - 6a_{n-2} + 3n, \text{ given that } a_0 = 0 \text{ and } a_1 = 1.$$

[(MATH3221.4, MATH3221.5, MATH3221.6)(Analyse/IOCQ)]

6 + 6 = 12

Cognition Level	LOCQ	IOCQ	HOCQ
Percentage distribution	13.54	48.46	37.50