DESIGN & ANALYSIS OF ALGORITHMS
(CSE2201)

Time Allotted : 2½ hrs **Full Marks : 60**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 4 (four) from Group B to E, taking one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A

1. Answer any twelve: **12 × 1 = 12**

Choose the correct alternative for the following

(i) $T(n) = 2T(n/2) + \Theta(n)$
The solution of the above recurrence will be:
(a) $\Theta(nlgn)$ (b) $\Theta(n)$
(c) $\Theta(n^2)$ (d) $\Theta(lgn)$

(ii) If the rank of the pivot element is 5 and there are n elements in the array,
What would the size of the partitions in Quick sort be?
(a) 1 and n-1 (b) 4 and n-5
(c) 5 and n - 6 (d) n - 4 and 4

(iii) Which algorithm technique is used to generate Huffman coding?
(a) Divide Conquer (b) Dynamic Programming
(c) Approximation Algorithm (d) Greedy Technique.

(iv) The optimal result of 0-1 Knapsack problem can be found only by using
(a) Greedy Algorithm (b) Divide & Conquer Technique
(c) Dynamic Programming (d) Approximation Algorithm

(v) An undirected graph G with n vertices is represented by an adjacency matrix where all diagonal elements are 0 and all non-diagonal elements are 1. Which of the following is TRUE?
(a) G does not have a minimum spanning tree (MST) (b) G has a unique MST of cost (n - 1)
(c) G has multiple MSTs of cost (n - 1) (d) None of the above.

(vi) In the KMP algorithm for pattern matching, the time complexity of the prefix function Π (x) for a pattern P of length m is –
(a) O(m log m) (b) O (m)
(c) O(m log n) (d) O(log m)

(vii) Consider the following matrices with given dimensions
x1 is 4 x 6
x2 is 6 x 8
x3 is 8 x 4
x4 is 4 x 5
Which of the following multiplication order gives the optimal solution?
(a) ((x1x2)x3)x4 (b) (x1(x2x3))x4
(c) x1((x2x3)x4) (d) (x1x2)(x3x4)

(viii) The problem of finding the Hamiltonian cycle of a graph can be solved in Polynomial time by
(a) Greedy method (b) Dynamic Programming
(c) Divide and conquer (d) None of the mentioned

(ix) The Disjoint Set data structure is essential to implement which algorithm?
(a) Prim's Algorithm (b) Bellman Ford Algorithm
(c) Kruskal Algorithm (d) Floyd Warshall Algorithm

(x) Which of the following problems is polynomial-time solvable?
(a) Longest Path problem (b) Finding a Hamiltonian circuit in a given graph
(c) Finding an Eulerian circuit in a given graph (d) None of the above.

Fill in the blanks with the correct word

(xi) If you want to prove that a problem P is NP-hard, then you need to show that, P can be reduced to any known NP-hard problem in polynomial time. The statement is _____ (True/ False)

(xii) The minimum height of an almost complete binary tree which represents a max-heap of 1023 elements will be _____.

(xiii) If T(n) = T(n/2) + n^2 represents a recurrence equation, then the solution of T (n) will be _____.

(xiv) Let G be a complete undirected graph on 4 vertices, having 6 edges with weights 1, 2, 3, 4, 5, 6. The maximum possible weight that a minimum weight spanning tree of G can have is _____.

(xv) The 0-1 knapsack problem is applied on n items with a sack having a capacity of W. The optimal result can be found at _____ index of the memoization table.

Group - B

2. (a) Prove that $g(n) = \Omega(f(n))$, iff $f(n) = O(g(n))$, where all the symbols have their standard meaning w.r.t asymptotic complexity. *[(CO3)(Apply/IOCQ)]*

(b) Asymptotically, how much time does insertion sort take when run on the following input: 2, 1, 4, 3, 6, 5, ..., n, n – 1? Give a brief justification for your answer. *[(CO4)(Analyse/IOCQ)]*

(c) What is the number of comparisons required to find the minimum and maximum of any four distinct numbers stored in any arbitrary order? Justify your answer.
Now, can you tell what is the number of comparisons required to sort any four numbers? Briefly justify your answer. *[(CO2)(Analyse/IOCQ)]*

(d) Is $2^{n+1} = O(2^n)$? Is $2^{2n} = O(2^n)$? Why? *[(CO2)(Understand/LOCQ)]*

3 + (1 + 2) + (2 + 2) + (1 + 1) = 12

3. (a) Consider the following recurrence:

$$T(n) = 2T(n/2) + \log n,\ T(1) = 0$$

Do you think Master Theorem can be applied to the above recurrence? If so, then apply the Master Theorem to solve the recurrence.
If not, then first write why it can't be applied and then apply the iterative method on the given recurrence to find the solution. *[(CO3)(Apply/IOCQ)]*

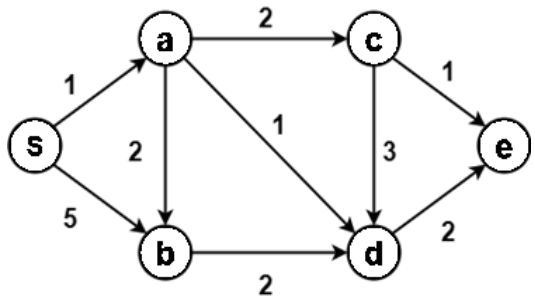
(b) Suppose you are given a set of n numbers $a_1, a_2, ..., a_n$. Prove that the number of comparisons required to sort these n numbers using any comparison-based sorting algorithm must be greater than or equal to: nlogn. *[(CO4)(Understand/LOCQ)]*

(2 + 5) + 5 = 12

Group - C

4. (a) Write the pseudo-codes for Initialize-Single–Source(G,s) and Relax(u,v,w) functions used in Dijkstra's algorithm to find out the shortest paths from a source vertex(s) to all other vertices of a given graph G(V, E). Assume that w is the non-negative weight of the edge(u, v). In the given graph G, there is a non-negative weight for each edge (u, v) ϵ E. *[(CO2)(Understand/HOCQ)]*

(b) Run Dijkstra's algorithm on the following graph G = (V, E) to find the shortest paths from source vertex 'S'. Show each step in detail.



(c) Write the optimal substructure property maintained by Prim's algorithm. *[(CO2)(Understand/IOCQ)]*

6 + 4 + 2 = 12

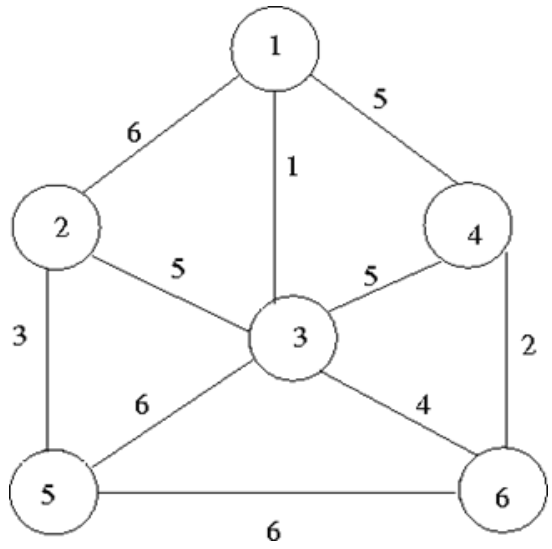
5. (a) A networking company uses a compression technique to encode the message before transmitting it over the network. Suppose the message contains the following distinct characters with their frequency (in thousands):

Character	Frequency
a	5
b	9
c	12
d	13
e	16
f	45

If the compression technique used is Huffman Coding, how many bits will be saved in the coding of the message compared with fixed-length coding of the same message? Please note that you are supposed to show all the steps to draw the tree corresponding to the Huffman code before giving the answer. *[(CO3)(Apply/IOCQ)]*

(b) Find out the Minimum Spanning Tree of the following graph using Prim's Algorithm with source vertex '2'.

[(CO3)(Apply/IOCQ)]



(c) If there is a negative weighted edge in a given graph, will Bellman Ford always be able to give a solution?

[(CO4)(Analyze/LOCQ)]

6 + 4 + 2 = 12

Group - D

6. (a) The capacity of a knapsack is 20. Which of the following item(s) should we take to maximize our profit? You can break any item and take it into the knapsack. Demonstrate your solution step-by-step. *[(CO3)(Apply/IOCQ)]*

Item	Profit	Weight
1	7	4
2	5	9
3	12	8

(b) Consider the problem of implementing a *k*-bit binary counter that counts upward from 0. We use an array A[0.. k – 1] of bits, where A.length = *k*, as the counter. A binary number *x* that is stored in the counter has its lowest-order bit in A[0] and its highest-order bit in A[k – 1], so that

$$x = \sum_{i=0}^{k-1} A[i].2^i$$

Initially, *x* = 0, and thus A[0] = 0 for i = 0, 1,...,k – 1. To add 1 (modulo 2^k) to the value in the counter, we use the following procedure. Calculate the amortized cost of each operation.

```
INCREMENT(A)
1    i = 0
2    while i<A.length and A[i] == 1
3        A[i] = 0
4        i = i + 1
5    if i<A.length
6        A[i] = 1
```

[(CO4)(Analyze/IOCQ)]

(c) Write an efficient algorithm to find the minimum spanning tree of a complete unweighted graph. Write down the time complexity of your algorithm. *[(CO2)(Understand/IOCQ)]*

4 + 5 + 3 = 12

7. (a) Suppose a skip-list of height h is created with the tossing of an unbiased coin, i, e., P(H) = ½.
(i) What is the probability that a particular element gets inserted in list S_i?
(ii) If there are n distinct items in the Skip-list, what is the expected size of the list S_i?
Finally show that the expected number of nodes in the Skip-list is 2n. *[(CO2,CO3,CO4)(Apply/IOCQ)]*

(b) Give the outline of an O(n^2) dynamic programming algorithm that computes T(n) defined as follows

$$T(n) = 2 \sum_{i=1}^{n-1} T(i)*T(i-1)$$
$$T(0) = T(1) = 2$$

[(CO2,CO3)(Apply/HOCQ)]

6 + 6 = 12

Group - E

8. (a) Write the approximation algorithm 'APPROX-VERTEX-COVER' for finding the Vertex Cover of a general graph and show that it is a polynomial-time 2-approximation algorithm. State its time complexity. *[(CO2,CO3)(Understand,Analyse/LOCQ)]*

(b) Explain the relation between the Clique Decision Problem, Maximum Independent Set Problem, and Vertex Cover Problem to show that if one of them is NP-hard, the other two have to be NP-hard. You may use a small example.

[(CO5)(Analyse/IOCQ)]

(3 + 2 + 1) + 6 = 12

9. (a) Define the following terms in the context of optimization problems -
(i) Polynomial time Approximation scheme (PTAS) (ii) FPTAS. *[(CO5)(Remember/LOCQ)]*

(b) State the Edge Cover Problem. What can you state about the computational complexity for this problem? *[(CO5)(Apply/IOCQ)]*

(c) Assume A and B are specific decision problems and that f is a polynomial-time function for reducing A to B. Which statement is true and which is false?

1. if $A \rightarrow_f B$ and $A \in P$, then $B \in P$
2. if $A \rightarrow_f B$ and $B \in NP$, then $A \in P$
3. if $A \rightarrow_f B$ and $A \in$ NP-Complete, then $B \in$ NP-Hard
4. if $A \rightarrow_f B$ and $B \in$ NP-Complete, then $A \in$ NP-Complete
5. if $A \rightarrow_f B$ and $B \rightarrow_f A$, then $A, B \in$ NP-Complete
6. if $A \rightarrow_f B$ and $B \in$ NP-Complete, then $B \rightarrow_f A$
7. if $A \rightarrow_f B$ and $A \in$ NP-Complete, then $B \in$ NP-Complete .

[(CO3, CO5)(Understand/LOCQ)]

$(1.5 \times 2) + 2 + 7 = 12$

Cognition Level	LOCQ	IOCQ	HOCQ
Percentage distribution	26.04	61.46	12.50