ADVANCED NUMERICAL METHODS
(MTH2202)

Time Allotted : 2½ hrs **Full Marks : 60**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 4 (four) from Group B to E, taking one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A

1. Answer any twelve: **12 × 1 = 12**

Choose the correct alternative for the following

(i) A system of equations $AX = b$ where $A = (a_{ij})_{n\times n}$ is said to be diagonally dominant if
(a) $|a_{ii}| \geq \sum_{\substack{j=1 \\ j\neq i}}^{n} |a_{ij}|$ for all i. (b) $|a_{ii}| < \sum_{\substack{j=1 \\ j\neq i}}^{n} |a_{ij}|$ for all i.
(c) $|a_{ii}| > \sum_{j=1}^{n} |a_{ij}|$ for all i. (d) $|a_{ii}| < \sum_{j=1}^{n} |a_{ij}|$ for all i.

(ii) In Gauss-Jordan method, the given system of linear equations represented by $AX = B$ is converted to another system $PX = Q$ where P is
(a) diagonal matrix. (b) identity matrix.
(c) upper triangular matrix. (d) lower triangular matrix.

(iii) Which of the following statements is NOT true about the singular values in the matrix Σ from the SVD?
(a) Singular values are always non-negative.
(b) Singular values are sorted in decreasing order.
(c) Singular values are the square roots of the eigenvalues of $A^T A$.
(d) Singular values can be negative.

(iv) The interval containing all the eigenvalues of the symmetric matrix $\begin{bmatrix} 3 & 2 & 2 \\ 2 & 5 & 2 \\ 2 & 2 & 3 \end{bmatrix}$ is
(a) $[-1, 7]$. (b) $[-1, 9]$.
(c) $[-7, 7]$. (d) $[1, 9]$.

(v) The value of $\Delta^3[(1-x)(1-2x)(1-3x)]$ taking $h = 1$ is
(a) 36. (b) 37. (c) -35. (d) -36.

(vi) Δe^x equals
(a) $e^{x+h} - e^x$ (b) $e^x - e^{x-h}$
(c) $e^x - e^{x+h}$ (d) $e^x(e^h - 1)$.

(vii) $[x, x_0, x_1]$ equals

(a) $\frac{[x,x_0]-[x_0,x_1]}{x-x_1}$ (b) $\frac{[x,x_0]-[x_0,x_1]}{x_1-x}$

(c) $\frac{[x_0,x_1]+[x,x_0]}{x+x_1}$ (d) $\frac{[x,x_1]-[x,x_0]}{x_1-x_0}$.

(viii) To generate the j^{th} experiment in Golden Section Search algorithm, the length of the j^{th} experiment L_j is given by

(a) $\frac{1}{\gamma^j}L_0$, (b) $\frac{1}{\gamma^{j+1}}L_0$, (c) $\frac{1}{\gamma^{j-1}}L_0$, (d) $\frac{1}{\gamma^{2j}}L_0$,

(ix) Out of the following algorithms, the one having the fastest rate of convergence is

(a) Dichotomous search. (b) Golden section search.

(c) Interval halving method. (d) Fibonacci search.

(x) In Fibonacci search, the search indices are determined

(a) by directly selecting the middle element in each step.

(b) by choosing the largest element in each subinterval.

(c) by using Fibonacci numbers to calculate the next indices in the sequence.

(d) by selecting random indices in the array.

Fill in the blanks with the correct word

(xi) If $A = GG^T$ is the Cholesky factorization of a symmetric positive definite matrix A, then G is a _____ matrix.

(xii) In the Gauss elimination method, if any one of the pivot elements is zero, then the method will _____.

(xiii) Newton backward interpolation formula is used for _____ intervals.

(xiv) If we go through r iterations, then the reduction ratio in Dichotomous Search algorithm is _____.

(xv) Geometrically, Simpson's one third rule for three points of interpolation represents a _____.

Group - B

2. (a) Is the following matrix positive definite? Justify your answer.

$$\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

[(MTH2202.1, MTH2202.4, MTH2202.6)(Understand/LOCQ)]

(b) Find the value of the infinity norm and the Euclidean norm of the matrix $A = \begin{bmatrix} 1 & 4 & 9 \\ 4 & 9 & 16 \\ 9 & 16 & 25 \end{bmatrix}$.

[(MTH2202.1, MTH2202.4, MTH2202.6)(Remember/LOCQ)]

(c) Find the solution of the given system of equations by Gauss Jacobi's method correct to 2 decimal places.

$$10x + y - z = 11.19$$
$$x + 10y + z = 28.08$$
$$-x + y + 10z = 35.61.$$

[(MTH2202.1, MTH2202.4, MTH2202.6)(Apply/IOCQ)]

2 + 4 + 6 = 12

3. (a) Solve the following system of equations by Cholesky's decomposition method:

$$x + 2y + 3z = 20$$
$$2x + 8y + 22z = 15$$
$$3x + 22y + 82z = 5.$$

[(MTH2202.1, MTH2202.4, MTH2202.6)(Evaluate/HOCQ)]

(b) Find the inverse of the following matrix, using Gauss-Jordan method:

$$\begin{bmatrix} 2 & 1 & -1 \\ 1 & 0 & -1 \\ 1 & 1 & 2 \end{bmatrix}.$$

[(MTH2202.1, MTH2202.4, MTH2202.6)(Apply/IOCQ)]

7 + 5 = 12

Group - C

4. (a) Find the QR decomposition of $A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \\ 3 & 4 \end{bmatrix}$. Hence, find the least squares solution of the system $Ax = b$, where $b = \begin{bmatrix} 5 \\ 8 \\ 11 \end{bmatrix}$. *[(MTH2202.3, MTH2202.4, MTH2202.6)(Analyse/IOCQ)]*

(b) Sketch the Gerschgorin's circles to estimate the bounds for the eigenvalues of the matrix $\begin{bmatrix} 6 & 2 & 1 \\ 2 & 5 & 3 \\ 1 & 3 & 4 \end{bmatrix}$. Shade and mention the smallest region containing all the eigenvalues of the given matrix. *[(MTH2202.3, MTH2202.4, MTH2202.6)(Understand/LOCQ)]*

8 + 4 = 12

5. Find the singular values, and hence the Singular Value Decomposition of the matrix $\begin{bmatrix} 3 & -1 \\ 1 & 3 \\ 1 & 1 \end{bmatrix}$. *[(MTH2202.3, MTH2202.4, MTH2202.6)(Evaluate/HOCQ)]*

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Group - D

6. (a) Evaluate $\int_0^1 \frac{1}{1+x}\,dx$ by using Simpson's $\frac{1}{3}^{rd}$ rule, taking eleven ordinates, and hence find the value of $\log_e 2$ correct up to five significant figures.

[(MTH2202.2, MTH2202.6)(Understand/LOCQ)]

(b) Find the value of $\sqrt{2}$ using Newton forward interpolation formula for the given data:

x	1.9	2.1	2.3	2.5	2.7
$f(x)$	1.3784	1.4491	1.5166	1.5811	1.6432

[(MTH2202.2, MTH2202.6)(Apply/IOCQ)]

6 + 6 = 12

7. (a) A curve passes through the points given by the following table:

x	1	1.5	2	2.5	3	3.5	4
y	2	2.4	2.7	2.8	3	2.6	2.1

Using Weddle's rule, find the area bounded by the curve, the x-axis and the lines $x = 1, x = 4$. *[(MTH2202.2, MTH2202.6)(Understand/LOCQ)]*

(b) The table gives the distance in nautical miles, of the visible horizon for the given heights m feet above the earth's surface:

x (height)	100	150	200	250	300	350	400
y (distance)	10.63	13.03	15.04	16.81	18.42	19.96	21.27

Use Newton's forward interpolation formula to find the value of y where $x = 218\ ft$. *[(MTH2202.2, MTH2202.6)(Apply/IOCQ)]*

6 + 6 = 12

Group - E

8. (a) Apply the principle of least squares to fit a straight line to the following data:

x	2	4	6	8	10	12	14
y	10	14	15	16	15	17	18

[(MTH2202.5, MTH2202.6)(Apply/IOCQ)]

(b) Use the Golden Section Search technique to maximize the function $f(x) = -3x^2 + 21.6x + 1$ in the interval $[0, 25]$ taking tolerance to be less than 1.0. *[(MTH2202.5, MTH2202.6)(Apply/IOCQ)]*

5 + 7 = 12

9. Use Dichotomous Search algorithm to minimize $f(x) = x^4 - 14x^3 + 60x^2 - 70x$ over $[0, 2]$ using tolerance 0.3. Consider $\epsilon = 0.001$. *[(MTH2202.5, MTH2202.6)(Apply/IOCQ)]*

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Cognition Level	LOCQ	IOCQ	HOCQ
Percentage distribution	22.92	57.29	19.79