MATHEMATICS – II
(MTH1201)

Time Allotted : 2½ hrs **Full Marks : 60**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 4 (four) from Group B to E, taking one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A

1. Answer any twelve: **12 × 1 = 12**

Choose the correct alternative for the following

(i) A random variable X has the following pdf :

$$f(x) = \begin{cases} k, & -2 < x < 1 \\ 0, & elsewhere \end{cases},$$

then value of constant k is

(a) $\frac{1}{8}$ (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) $\frac{1}{12}$.

(ii) If X is normally distributed with zero mean and unit variance, then $E(X^2)$ is

(a) 0 (b) 1 (c) 2 (d) -1

(iii) One of the roots of $x^2 + 5x - 3 = 0$ lies between

(a) 0 and 1 (b) 1 and 2 (c) 2 and 3 (d) 3 and 4.

(iv) Modified Euler method of order four is analogous with Taylor series expansion up to first

(a) 2 terms (b) 3 terms (c) 4 terms (d) 5 terms.

(v) A binary tree has exactly

(a) two vertices of degree two (b) one vertex of degree two
(c) one vertex of degree one (d) two vertices of degree one.

(vi) The number of pendant vertices in a binary tree having n vertices

(a) $\frac{n+1}{2}$ (b) $\frac{n-1}{2}$ (c) $\frac{n(n+1)}{2}$ (d) n.

(vii) The value of $\Gamma\left(\frac{3}{4}\right)\Gamma\left(\frac{1}{4}\right)$ is

(a) $\sqrt{2\pi}$ (b) $\sqrt{2}\pi$ (c) $2\sqrt{\pi}$ (d) $\frac{\pi}{\sqrt{2}}$.

(viii) $\mathcal{L}\{e^{-2t}\cos t\}$ is

(a) $\frac{s+2}{s^2+4s+5}$ (b) $\frac{s}{s^2+4s+5}$ (c) $\frac{s+1}{s^2+4s+1}$ (d) $\frac{s+3}{s^2+4s+5}$

(ix) The value of $\beta\left(-\frac{3}{2},\frac{7}{2}\right)$ is

(a) $-\frac{5\pi}{2}$ (b) $\frac{5\pi}{2}$ (c) $\frac{5\pi}{4}$ (d) $\sqrt{\pi}$.

(x) If the number 37.46235 rounded off to four significant digits then the percentage of error is

(a) 6.27×10^{-3} (b) 6.27×10^{-5}

(c) 6.27×10^{-2} (d) 6.27×10^{3}.

Fill in the blanks with the correct word

(xi) If a binary tree has 20 pendant vertices, then the number of internal vertices of the tree is _____.

(xii) $L^{-1}\left(\frac{24}{(p+1)^5}\right) =$ _____.

(xiii) In the LU factorization method, a matrix A can be factorized into $A = LU$, where L is _____.

(xiv) Rounding off the number 0.004935 to 3 significant figures, we get _____.

(xv) 5 boys and 3 girls are seated randomly in a row. The probability that no boy sits between 2 girls is _____.

Group - B

2. (a) There are two identical urns containing 4 white, 3 red balls and 3 white 7 red balls respectively. An urn is chosen at random and a ball is drawn from it. If the ball drawn is white, then what is the probability that it is from the first urn?

[(MTH1201.1, MTH1201.2)(Understand /LOCQ)]

(b) Marks obtained by 1000 students to a final examination are found to be normally distributed with mean 70 and standard deviation 5. Estimate the number of students whose marks will be

(i) between 60 and 79, both inclusive.

(ii) below 40.

[(MTH1201.1, MTH1201.2) (Evaluate /HOCQ)]

6 + 6 = 12

3. (a) Determine the value of k such that

$$f(x) = \begin{cases} k\,x(1-x), & 0 < x < 1 \\ 0, & elsewhere \end{cases}$$

is a possible p. d. f of a continuous random variable X. Determine the distribution function of X and $E(X)$.

[(MATH1201.1, MATH1201.2) (Remember/LOCQ)]

(b) Suppose that an airplane engine will fail, when in flight, with probability $1-p$ independently from engine to engine; suppose that the airplane will make a successful flight if at least 50 percent of its engines remain operative. For what values of p is a four-engine plane preferable to a two-engine plane?

[(MATH1201.1, MATH1201.2)(Analyse/IOCQ)]

6 + 6 = 12

Group - C

4. (a) Solve the following system of equations using Gauss-Seidel method.

$6x + 15y + 2z = 72$

$x + y + 54z = 110$

$27x + 6y - z = 85.$ *[(MTH1201.3)(Apply/IOCQ)]*

(b) Find a real root of the equation $x^3 - 2x - 5 = 0$ using Regula-Falsi method correct to three places of decimal. *[(MTH1201.3)(Evaluate/HOCQ)]*

6 + 6 = 12

5. (a) Using Runge – Kutta method of fourth order, find $y(1.4)$ for $\frac{dy}{dx} = 3x + y^2$, $y(1) = 1$ by taking $h = 0.2$. *[(MATH1201.3)(Apply/IOCQ)]*

(b) Using Newton-Raphson method evaluate $\sqrt[5]{3}$, correct upto four decimal places. *[(MATH1201.3)(Apply/IOCQ)]*

6 + 6 = 12

Group - D

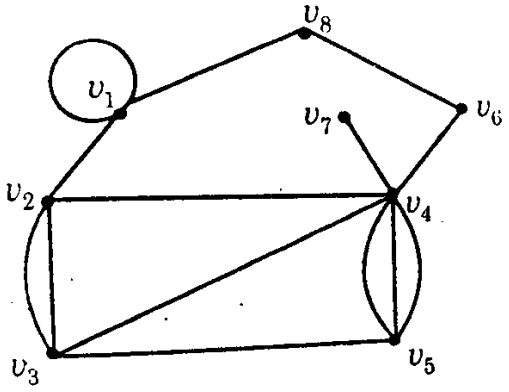
6. (a) Prove that the number of edges in a simple graph with n vertices cannot exceed $\frac{n(n-1)}{2}$. *[(MTH1201.4)(Remember/LOCQ)]*

(b) Find the graph whose incidence matrix is the following:

$$\begin{bmatrix} 1 & -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & -1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & -1 & -1 & 0 \end{bmatrix}$$

[(MTH1201.4)(Understand/LOCQ)]

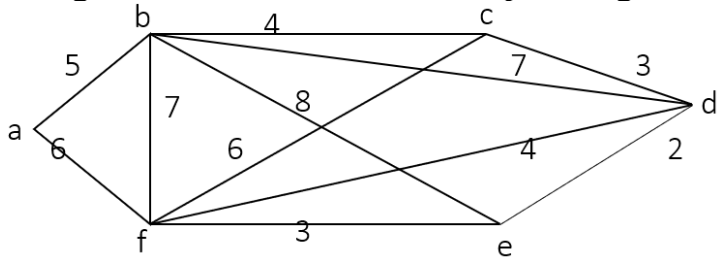
(c) Find a spanning tree of the following graph by BFS algorithm.



[(MTH1201.4)(Apply/IOCQ)]

4 + 4 + 4 = 12

7. (a) Use Kruskal's algorithm to find a minimal spanning tree of the following graph.



[(MTH1201.4)(Apply/IOCQ)]

(b) Show that the number of vertices in a binary tree is always odd.
[(MTH1201.4)(Understand/LOCQ)]

(c) Find the minimum and maximum no. of edges of a simple graph with ten vertices and 3 components. *[(MTH1201.4)(Understand/LOCQ)]*

6 + 4 + 2 = 12

Group - E

8. (a) Evaluate $\int_0^\infty \frac{e^{-at}-e^{-bt}}{t}dt$ *[(MTH1201.5, MTH1201.6)(Evaluate/HOCQ)]*

(b) Using Laplace Transform, find the solution of the initial value problem
$y'' - 3y' + 2y = 4t + e^{3t}$, given that $y(0) = 1$ and $y'(0) = -1$
[(MTH1201.5, MTH1201.6)(Apply/IOCQ)]

6 + 6 = 12

9. (a) Evaluate the integral $\int_0^\infty te^{-3t}sint\ dt$. *[(MTH1201.5, MTH1201.6)(Evaluate/HOCQ)]*

(b) Find $\mathcal{L}^{-1}\left\{\frac{3s-2}{s^2-4s+20}\right\}$. *[(MTH1201.5, MTH1201.6)(Evaluate/HOCQ)]*

(c) Prove that $\int_0^{\frac{\pi}{2}} \frac{d\theta}{\sqrt{sin\theta}} \times \int_0^{\frac{\pi}{2}} \sqrt{sin\theta}d\theta = \pi$. *[(MTH1201.5, MTH1201.6)(Understand/LOCQ)]*

4 + 4 + 4 = 12

Cognition Level	LOCQ	IOCQ	HOCQ
Percentage distribution	31.25	41.67	27.08