ADVANCED DISCRETE MATHEMATICS AND STATISTICAL METHODS
(MATH 5101)

Time Allotted : 2½ hrs   **Full Marks : 60**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and
<u>any 4 (four)</u> from Group B to E, taking <u>one</u> from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A

1. Answer any twelve: **12 × 1 = 12**

Choose the correct alternative for the following

(i) The probability of obtaining point one or an even point on the top face in the case of a rolling of a die once is
(a) $\frac{1}{6}$ (b) $\frac{1}{2}$ (c) $\frac{2}{3}$ (d) $\frac{1}{4}$

(ii) Two coins are tossed. The probability of getting at least one head is
(a) $\frac{3}{4}$ (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) $\frac{2}{3}$

(iii) The variance of a random variable X is
(a) $\{E(X)\}^2$ (b) $E(X^2)$
(c) $E(X^2) - \{E(X)\}^2$ (d) $E(X^2) - E(X)$

(iv) The mean, median, mode of the data $0, 5, -1, 5, 2, 3, -1, 4, 3, 0, 0, 3, 3$ are
(a) $2, 3, 3$ (b) $2, 2, 3$ (c) $3, 3, 3$ (d) None

(v) The mean of the Poisson distribution with parameter μ is
(a) μ (b) μ^2 (c) $-\mu$ (d) $-\mu^2$

(vi) The sum of the coefficients in the expression of $(x + y + z)^{10}$ is
(a) 2^{10} (b) 2^9 (c) 3^{10} (d) 3^9

(vii) The greatest common divisor of two consecutive Fibonacci numbers is
(a) 3   (b) 1   (c) 2   (d) 4

(viii) The number of ways in which 8 people can sit around a table is
(a) 5040   (b) 40320   (c) 720   (d) 120

(ix) The chromatic number of a tree with six vertices is
(a) 6   (b) 7   (c) 3   (d) 2

(x) The dual of K_3, the complete graph having 3 vertices, has
(a) 3 vertices   (b) 3 edges
(c) 2 edges   (d) 4 vertices

Fill in the blanks with the correct word

(xi)   The positive square root of the variance of a random variable X is called the _____ of X.

(xii)   The number of possible mobile phone numbers of 10 digits is _____.

(xiii)   A die is tossed. If the number that turns up is odd, the probability that it is prime is _____.

(xiv)   The number of edges in the dual of a simple planar graph having 6 edges _____.

(xv)   The chromatic polynomial of a tree having 8 vertices is _____.

Group - B

2. (a) A and B are two independent witnesses. The probability that A will speak the truth is x and the probability that B will speak the truth is y. A and B agree in a certain statement. Show that the probability that the statement is true is $\frac{xy}{1-x-y+2xy}$.   *[(MATH 5101.1, MATH5101.2)(Analyse/IOCQ)]*

    (b)   If 4 balls are drawn (i) with replacement or (ii) without replacement from an urn containing 8 white and 3 black balls, then find the expectation of the number of white balls in the cases (i) and (ii).   *[(MATH 5101.1, MATH5101.2)(Understand/LOCQ)]*

    6 + 6 = 12

3. (a)   If A and B are independent events, then show that the following pairs are also independent:
    (i) $\bar{A}$ and $\bar{B}$, (ii) A and $\bar{B}$, (iii) $\bar{A}$ and B.   *[(MATH 5101.1, MATH5101.2)(Remember/LOCQ)]*

    (b)   In a bolt factory, machine A, B, C manufacture respectively 25%, 35% and 40% of the total. Of their output 5%, 4% , and 2% are defective bolts. A bolt is drawn at random from their product and is found to be defective. What are the probabilities that it was manufactured by machine A, B and C.
    [(MATH 5101.1, MATH5101.2)(Evaluate/HOCQ)]

    6 + 6 = 12

Group - C

4. (a)   The overall percentage of failures in a certain examination is 40. What is the probability that out of a group of 6 candidates at least 4 passed the examinations?
    [(MATH5101.1,MATH5101.2)(Analyse/IOCQ)]

    (b)   The mean of a normal distribution is 50 and 5% of the values are greater than 60. Find the standard deviation of the distribution (Given that the area under the standard normal curve between $z = 0$ and $z = 1.64$ is 0.45).
    [(MATH5101.1,MATH5101.2)(Remember/LOCQ)]

    6 + 6 = 12

5. (a)   Out of two regression lines given by $x + 4y + 3 = 0$ and $4x + 9y + 5 = 0$, which one is the regression line of y on x ? Find the mean of x and mean of y. Also find the correlation coefficient between x and y. Estimate the value of x when $y = 1.4$.
    [(MATH5101.1,MATH5101.2)(Evaluate/HOCQ)]

(b) The table below gives the marks obtained in a test in mathematics.

Marks (x)	$1 - 10$	$11 - 20$	$21 - 30$	$31 - 40$	$41 - 50$	$51 - 60$
No of Students	3	16	26	31	16	8

Calculate the mean and standard deviation of the distribution.

[(MATH5101.1,MATH5101.2)(Remember/LOCQ)]

6 + 6 = 12

Group - D

6. (a) Determine the number of subsets of a set with n elements. Justify your answer in detail. *[(MATH5101.1,MATH5101.2,MATH5101.3)(Understand/IOCQ)]*

(b) In how many ways can 7 women and 3 men be arranged in a row if the 3 men must always stand next to each other? Show your work in detail.

[(MATH5101.1,MATH5101.2,MATH5101.3)(Analyse/IOCQ)]

6 + 6 = 12

7. (a) Solve the following recurrence relations by the method of characteristic roots:
(i) $a_n - 7a_{n-1} + 12a_{n-2} = 0$, for $n \geq 2$, where $a_0 = 2, a_1 = 5$. (ii) $a_n - 5a_{n-1} + 6a_{n-2} = 0$, for $n \geq 2$, where $a_0 = 2, a_1 = 5$.

[(MATH5101.1,MATH5101.2,MATH5101.3)(Apply/IOCQ)]

(b) Using the pigeonhole principle, prove that, given any set of 7 distinct integers, there must exist 2 integers in this set whose sum or difference is divisible by 10.

[(MATH5101.1,MATH5101.2,MATH5101.3)(Apply/IOCQ)]

6 + 6 = 12

Group - E

8. (a) Prove that a simple connected planar graph G having n vertices and e edges determines $f = e - n + 2$ regions. *[(MATH5101.1,MATH5101.2,MATH5101.4)(Remember/LOCQ)]*

(b) Let G be a simple connected planar graph with n vertices, e edges and f regions. Then (i) $e \geq \frac{3}{2}f$, (ii) $e \leq 3n - 6$. *[(MATH5101.1,MATH5101.2,MATH5101.4)(Remember/LOCQ)]*

6 + 6 = 12

9. (a) (i) Prove that K_4, the complete graph having four vertices, is planar. (ii) Prove that K_6, the complete graph having six vertices, is non-planar. State any theorem that you use. State any theorem that you use.

[(MATH5101.1,MATH5101.2,MATH5101.4)(Analyse/IOCQ)]

(b) Prove that a bipartite graph cannot contain a cycle of odd length.

[(MATH5101.1,MATH5101.2,MATH5101.4)(Analyse/IOCQ)]

6 + 6 = 12

Cognition Level	LOCQ	IOCQ	HOCQ
Percentage distribution	37.5	50	12.5