ADVANCED DATA STRUCTURES
(CSEN 5101)

Time Allotted : 2½ hrs **Full Marks : 60**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 4 (four) from Group B to E, taking one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A

1. Answer any twelve: **12 × 1 = 12**

Choose the correct alternative for the following

(i) The time complexity of Heapify operation on an array of n elements is
(a) O(log n) (b) O(n) (c) O(n * log n) (d) $O(n^2)$

(ii) Which one of the following statements is false?
(a) In a full binary tree, each node is either a leaf or has degree exactly equal to 2
(b) A binary tree is simply an ordered tree in which each node has degree at most 2
(c) A binary tree is a k-ary tree with k = 2
(d) Total number of internal nodes in a complete k-ary tree of height h is $(k^h-1)/(k-1)$.

(iii) Consider the following postfix expression with single digit operands, where the operators have their usual meaning:

8 2 3 ^ / 2 3 * + 5 1 * -

Which of the following correctly represents the top two elements of the stack just after the second * is evaluated?
(a) 6, 1 (b) 5, 7 (c) 1, 5 (d) 3, 2

(iv) In a binary tree, the number of internal nodes of degree 1 is 5 and the number of internal nodes of degree 2 is 10. The number of leaf nodes in the binary tree is
(a) 10 (b) 11 (c) 12 (d) 13.

(v) The inorder and preorder traversal of a binary tree results are given below.
Inorder: 42, 29, 51, 10, 37, 63
Preorder: 10, 29, 42, 51, 37, 63
The postorder traversal result is
(a) 29, 51, 10, 37, 63, 42 (b) 51, 10, 37, 63, 42, 29
(c) 42, 51, 29, 63, 37, 10 (d) 10, 37, 63, 42, 29, 51.

(vi) The minimum number of children of all non-leaf nodes except the root in a B-tree of order m is
(a) floor(m/2) (b) ceiling(m/2)
(c) floor(1 + m/2) (d) ceiling(1 + m/2)

(vii) The minimum possible number of nodes that can be present in an AVL tree of height 4 is
(a) 13 (b) 12 (c) 11 (d) 10.

(viii) The maximum number of items in a B-tree of order 5 and height 3 is
(a) 124 (b) 126 (c) 624 (d) 626.

(ix) Let Q = {p_1, p_2, p_3, p_4} be a set of 4 points in the X-Y plane. In which of the following cases the line segment joining p_1 and p_2 does not intersect the line segment joining p_3 and p_4?
(a) The segments p_1p_2 and p_3p_4 straddle each other's lines
(b) The signs of the cross products $(p_1-p_3) \times (p_4-p_3)$ and $(p_2-p_3) \times (p_4-p_3)$ are different
(c) p_3 is colinear with p_1p_2 but it is not between p_1 and p_2
(d) p_3 is colinear with p_1p_2 and it is between p_1 and p_2.

(x) It is required to search for a given string P of n characters in another string T of m characters, where n is much smaller than m. If a naïve (brute-force) pattern matching method is used, the search will take a worst-case time roughly proportional to
(a) m (b) n (c) m * n (d) m + n

Fill in the blanks with the correct word

(xi) A heap can support any priority queue operation on a set of size n in O(_____) time.

(xii) If a hash table is 25% full, its load factor is _____.

(xiii) A balanced BST, storing n points can be built in O(_____) time, if the points are given in sorted order.

(xiv) A positive integer n, when expressed in octal (i.e., radix 8) notation has 12 octal digits with no leading zeroes. If n is now expressed in hexadecimal (i.e., radix 16) notation, the number of hexadecimal digits will be _____.

(xv) The prefix pattern of pattern matching is represented by π. The value of π(2) for the pattern "aaa" is _____.

Group - B

2. (a) Write the pseudo code of Increase_Key function for a max priority queue.
[(CO2)(Understand/LOCQ)]

(b) Consider the max heap represented by the following array: 40, 30, 20, 10, 15, 16, 17, 8, 4
What will be the array representation of the heap after a new value 35 is inserted? Show all the intermediate steps. *[(CO2)(Understand/LOCQ)]*

(c) An array contains the numbers as shown below. Show the steps to sort these numbers in ascending order, using heapsort algorithm.

20	15	3	10	2	1	7

[(CO2)(Apply/IOCQ)]
3 + 4 + 5 = 12

3. Given input {4371, 1323, 6173, 4199, 4344, 9679, 1989} and a Hash function h(X) = X mod 10. Show the results in
   (i) separate chaining hash table
   (ii) open addressing hash table using linear probing
   (iii) open addressing hash table using quadratic probing
   (iv) open addressing hash table with 2nd hash function $h_2(X) = 7 - (X \bmod 7)$.

[(CO3)(Analyse/HOCQ)]

(4 × 3) = 12

Group - C

4. (a) What is a Binary Search Tree (BST)? Write a pseudo code to display all the keys in a BST in sorted order. *[(CO3)(Understand and Apply/IOCQ)]*

   (b) Showing each step, construct the binary tree whose preorder and postorder sequences are given below:
   Preorder: F, B, A, D, C, E, G, I, H
   Postorder: A, C, E, D, B, H, I, G, F *[(CO2)(Analyse/IOCQ)]*

   (c) Same set of values are represented by two BSTs T_1 and T_2 with heights H_1 and H_2 respectively ($H_1 > H_2$). Is T_1 more efficient than T_2? Explain. *[(CO2)(Analyse/HOCQ)]*

(2 + 3) + 5 + 2 = 12

5. (a) (i) Show the necessary steps of constructing a binary search tree using the following key elements according to the given order of insertion: 20, 15, 30, 5, 17, 25, 37, -5, 7, 23, 33, 45
   (ii) Now, delete the following keys in sequence from the tree: 7, 20, 15, 37, 30, 5. *[(CO3)(Analyse/IOCQ)]*

   (b) Derive the formula for the number of all possible BSTs with N distinct elements. *[(CO3)(Apply/IOCQ)]*

(4 + 4) + 4 = 12

Group - D

6. (a) Define B-Tree. Explain one scenario where using a B-tree is beneficial. *[(CO3)(Understand/LOCQ)]*

   (b) Show the B-tree that results when inserting the keys R, Y, F, X, A, M, C, D, E, T, H, V, L, W, G (in the given sequence using lexicographic ordering). Assume a minimum branching factor of t = 3. Show the B-tree just before and after a node split as keys get inserted. *[(CO3)(Analyse/IOCQ)]*

(2 + 3) + 7 = 12

7. (a) Given a sorted list of n keys, how do we construct a skip list that ensures the average time to search a key in the list is proportional to lg n? Use the following sorted list of 21 keys to illustrate the construction method: 21, 25, 33, 39, 46, 51, 56, 62, 68, 73, 81, 85, 89, 92, 97, 101, 110, 115, 118, 125, 134. *[(CO3)(Analyse/HOCQ)]*

   (b) Show the necessary steps of constructing a Red-Black Tree using the following key elements according to the given order of insertion: 8, 18, 5, 15, 17, 25, 40, 80. *[(CO3)(Analyse/HOCQ)]*

6 + 6 = 12

Group - E

8. (a) How is one dimensional range searching performed? Analyse its time complexity.
[(CO5)(Analyse/IOCQ)]

(b) Given the following table.

Character	Frequency
A	5
B	9
C	12
D	13
E	16
F	45

Show all the steps to find out the Huffman code for each of the characters.
[(CO5)(Apply/IOCQ)]
(3 + 3) + 6 = 12

9. (a) Draw the k-D tree that results from inserting the following points A(2, 3), B(4, 2), C(4, 5), D(3, 3), E(1, 5), F(4, 4), and G(1, 1). Show each intermediate step.
[(CO3)(Analyse/IOCQ)]

(b) Show the necessary steps and data of searching the pattern "ATATCG" in the text "GCATCCATATCCATATCGATCG", using Knuth Morris Pratt Algorithm.
[(CO5)(Apply/IOCQ)]

(c) Calculate the number of bits that may be required for encoding the message 'abccbccbaab' deploying Huffman encoding. *[(CO5)(Apply/IOCQ)]*
4 + 5 + 3 = 12

Cognition Level	LOCQ	IOCQ	HOCQ
Percentage distribution	12.50	60.42	27.08