INTRODUCTION TO OPTIMIZATION
(MTH2104)

Time Allotted : 2½ hrs **Full Marks : 60**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 4 (four) from Group B to E, taking one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A

1. Answer any twelve: **12 × 1 = 12**

Choose the correct alternative for the following

(i) To convert ≥ type constraints into equality constraints, we must
(a) add a surplus variable
(b) subtract an artificial variable
(c) subtract a surplus variable and add an artificial variable
(d) add a surplus variable and subtract an artificial variable.

(ii) The possible number of basic solutions in a system of m equations in n unknowns are
(a) n (b) m (c) mn (d) $\binom{n}{m}$.

(iii) The value of objective function is maximum under linear constraints
(a) at the centre of feasible region (b) at (0,0)
(c) at any vertex of feasible region (d) the vertex which is at maximum distance from $(0, 0)$.

(iv) The solution of a transportation problem with m rows and n columns is feasible if the number of positive allocations are
(a) mn (b) $m + n$ (c) $m + n - 1$ (d) $m + n + 1$.

(v) If there were n workers and n jobs in an assignment problem, there would be
(a) $(n - 1)!$ solutions (b) $n!$ solutions
(c) $(n + 1)!$ solutions (d) n solutions.

(vi) The branch-and-bound method is used to solve
(a) LP problems (b) NLP problems (c) GP problems (d) IP problems.

(vii) If $\underline{V} \neq \overline{V}$, the game is said to be
(a) non-strictly determinable (b) strictly determinable
(c) biased (d) fair game.

(viii) In a PERT network the starting vertex is a
(a) burst node (b) merge node (c) root (d) none of these.

(ix) A saddle point exists when
(a) maximin value = maximax value (b) minimax value = maximax value
(c) minimax value = maximin value (d) minimax value ≠ maximin value.

(x) The critical path
(a) is a path that operates from the starting node to the end node
(b) is a mixture of all paths
(c) is the longest path
(d) is the shortest path.

Fill in the blanks with the correct word

(xi) Artificial variable is use in _____ type of inequality.

(xii) In big M method, the simplex algorithm automatically discards the _____ variable from the solution.

(xiii) When total supply is equal to total demand in a transportation problem, the problem is said to be _____ problem.

(xiv) In Game theory, if a player A selects a particular course of action, it is said to be a _____ strategy.

(xv) If there are n jobs to be processed through two machines $M_1$and M_2 in order M_1M_2 then the idle time for machine $M_1$is _____.

Group - B

2. (a) Solve the following LP problem using Big-M method.
Maximize $Z = 2x_1 + x_2 + 3x_3$

Subject to Constraints:

$2x_1 + 3x_2 + 4x_3 = 12$

$x_1 + x_2 + 2x_3 \leq 5$

$x_1, x_2, x_3 \geq 0$ *[(MTH2104.1, MTH2104.4)(Evaluate/HOCQ)]*

(b) Write down the dual of the following LP problem.

Minimize $Z = x_1 + x_2 + x_3$

Subject to Constraints:

$x_1 - 3x_2 + 4x_3 = 5$

$x_1 - 2x_2 \leq 3$

$2x_2 - x_3 \geq 4$

$x_1, x_2 \geq 0, x_3$ unrestricted in sign *[(MTH2104.1, MTH2104.4)(Understand/LOCQ)]*

8 + 4 = 12

3. (a) Prove that $B = \{(x_1, x_2): x_1^2 + x_2^2 \leq 9\}$ is convex set. *[(MTH2104.1, MTH2104.4)(Understand/LOCQ)]*

(b) Use the Simplex method to find the maximum value of

$Z = 3x_1 + x_2 + 3x_3,$

Subject to Constraints:

$2x_1 + x_2 + x_3 \leq 2,$

$x_1 + 2x_2 + 3x_3 \leq 5,$

$2x_1 + 2x_2 + x_3 \leq 6,$

where $x_1, x_2, x_3 \geq 0.$ *[(MTH2104.1, MTH2104.4)(Evaluate/HOCQ)]*

4 + 8 = 12

Group - C

4. (a) A lead draftsman has five drafting tasks to accomplish and five idle draftsmen. Each draftsman is estimated to require the following number of hours for each task.

		Tasks				
		A	B	C	D	E
Draftsman	1	60	50	100	85	95
	2	65	45	100	75	90
	3	70	60	110	97	85
	4	70	55	105	90	95
	5	60	40	120	85	97

If each draftsman costs the company Rs.15.80 per hour, including overhead, find the assignment of draftsmen to tasks that will result in the minimum total cost. What would be the total cost? *[(MTH2104.1, MTH2104.2)(Apply/IOCQ)]*

(b) Explain that "assignment problem is a special case of transportation problem". *[(MTH2104.1, MTH2104.2)(Understand/LOCQ)]*

9 + 3 = 12

5. A company has three production facilities S1, S2 and S3 with production capacity of 7, 9 and 18 units (in 100s) per week of a product, respectively. These units are to be shipped to four warehouses D1, D2, D3 and D4 with requirement of 5, 6, 7 and 14 units (in 100s) per week, respectively. The transportation costs (in rupees) per unit between factories to warehouses are given in the table below:

	D1	D2	D3	D4	Supply (Availability)
S1	19	30	50	10	7
S2	70	30	40	60	9
S3	40	8	70	20	18
Demand (Requirement)	5	8	7	14	

(i) Formulate the transportation problem as an LP model to minimize the total transportation cost. *[(MTH2104.1, MTH2104.2)(Understand/LOCQ)]*

(ii) Find an initial basic feasible solution to the transportation problem using Vogel's Approximation Method (VAM). *[(MTH2104.1, MTH2104.2)(Analyse/IOCQ)]*

(6 + 6) = 12

Group - D

6. (a) Solve the following game using graphical method. *[(MTH2104.1, MTH2104.3)(Apply/IOCQ)]*

		Player B		
Player A		4	-1	0
		-1	4	2

(b) For what value of a, the game with the following payoff matrix is strictly determinable? *[(MTH2104.1, MTH2104.3)(Analyse/IOCQ)]*

		B		
		I	II	III
A	I	a	5	2
	II	-1	a	-8
	III	-2	3	a

6 + 6 = 12

7. (a) Find the ranges of value of P and Q which will render the entry (2,2) a saddle point for the following game.

[(MTH2104.1, MTH2104.3)(Analyse/IOCQ)]

		Player B	
Player A	2	4	5
	10	7	Q
	4	P	6

(b) Use algebraic method to solve the following game. *[(MTH2104.1, MTH2104.3)(Apply/IOCQ)]*

		Player B	
	x_1	-1	2	1
Player A	x_2	1	-2	2
	x_3	3	4	-3
		y_1	y_2	y_3

6 + 6 = 12

Group - E

8. (a) Consider the project with activity and duration given as follows:

Activity	**Immediate predecessor**	**Duration (Days)**
A	—	3
B	A	2
C	A	6
D	A	3
E	C, D	7
F	D	4
G	E	3
H	G	25
I	F, H	10
J	B, I	20

(i) Draw the network diagram for the project.
(ii) Identify the critical path.
(iii) What is the project duration?
(iv) Find out the total float associated with each activity. *[(MTH2104.1, MTH2104.5, MTH2104.6)(Understand/LOCQ)]*

(b) There are four jobs A, B, C and D, which are to be, processed on machines M_1, M_2, M_3 and M_4 in the order $M_1M_2M_3M_4$. The processing time in hours is given below. Find the optimal sequence as well as idle time of jobs, and waiting time for machines.

Machine	***Jobs***			
	A	*B*	*C*	*D*
M_1	15	12	13	16
M_2	5	2	3	0
M_3	4	10	6	3
M_4	14	12	15	19

[(MTH2104.1, MTH2104.5, MTH2104.6)(Analyse/IOCQ)]

(3 + 2 + 1 + 2) + 4 = 12

9. (a) Five jobs are performed, first on machine X and then on machine Y. The time taken, in hours by each job on each machine is given below:

Job :	A	B	C	D	E
Time on machine X :	12	4	20	14	22
Time on machine Y :	6	14	16	18	10

Determine the optimum sequence of jobs that minimizes the total elapsed time to complete the jobs. Also compute the minimum time. *[(MTH2104.1, MTH2104.5, MTH2104.6)(Analyse/IOCQ)]*

(b) We have five jobs, each of which must go through the machines A, B and C in the order ABC. Processing times (in hours) is as follows:

Job :	1	2	3	4	5
Machine A :	5	7	6	9	5
Machine B :	2	1	4	5	3
Machine C :	3	7	5	6	7

Determine the sequence for the jobs that will minimize the total elapsed time. *[(MTH2104.1, MTH2104.5, MTH2104.6)(Analyse/IOCQ)]*

6 + 6 = 12

Cognition Level	LOCQ	IOCQ	HOCQ
Percentage distribution	26.04	57.29	16.67