MATHEMATICAL FOUNDATIONS
(MCA1104)

Time Allotted : 2½ hrs **Full Marks : 60**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 4 (four) from Group B to E, taking one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A

1. Answer any twelve: **12 × 1 = 12**

Choose the correct alternative for the following

(i) A complete graph with five vertices is called
(a) regular graph (b) Kuratowski's First graph
(c) Kuratowski's Second graph (d) None of these

(ii) If p be the pendant vertices in a binary tree T with n vertices, then
(a) $p = \frac{n+1}{2}$ (b) $n = \frac{p+1}{2}$
(c) $p = \frac{n-1}{2}$ (d) $n = \frac{p-1}{2}$

(iii) The determinant value of a skew symmetric matrix of even order is
(a) a perfect square (b) always positive natural number
(c) zero (d) always negative integer.

(iv) An m× n matrix of rank r can be expressed as a product of 2 matrices each of rank
(a) $\frac{r}{2}$ (b) 2r (c) r-1 (d) r

(v) A card is drawn from a pack of 52 cards. The probability of getting a queen of club or a king of heart is
(a) 1/13 (b) 2/13 (c) 1/26 (d) 1/52

(vi) If X is normally distributed with mean 0 and variance 1, then the expectation of X^2 is
(a) 1 (b) 2 (c) 8 (d) 0

(vii) If T is an unbiased estimator of θ, then T^2 a biased estimator of
(a) θ^2 (b) $\sqrt{\theta}$ (c) θ (d) all of the above

(viii) The power of a test is defined as
(a) 1 – P(Type I Error) (b) 1 – P(Type II Error)
(c) P(Type I Error) P(Type II Error) (d) 1 – P(Type I Error) P(Type II Error)

(ix) A passing student is failed by an examiner, it is an example of
(a) Type-I error (b) Type-II error
(c) Best decision (d) All of the above

(x) The unbiased estimator of population variance σ^2 is
(a) $\frac{1}{n}\sum_{i=1}^{n}\frac{(x_i-\bar{x})^2}{n}$ (b) $\frac{1}{n-1}\sum_{i=1}^{n}\frac{(x_i-\bar{x})^2}{n}$
(c) $\frac{1}{n}\sum_{i=1}^{n}\frac{(x_i-\mu)^2}{n}$ (d) $\frac{1}{n-1}\sum_{i=1}^{n}\frac{(x_i-\mu)^2}{n}$

Fill in the blanks with the correct word

(xi) Every skew symmetric determinant of odd order is equal to _____.

(xii) The number of edges in a regular graph of degree 46 and 8 vertices is _____.

(xiii) The regression equation of y on x is _____.

(xiv) The point estimator of population mean μ is _____.

(xv) H_0: $\theta \neq 0$ is a _____ hypothesis (simple/composite).

Group - B

2. (a) Show that a connected graph with n vertices and n-1 edges is a tree.

[(CO1) (Analyze/IOCQ)]

(b) State Kruskal algorithm for finding minimum spanning-tree. With the help of that, find the minimum spanning-tree for the following graph in Fig. 1.

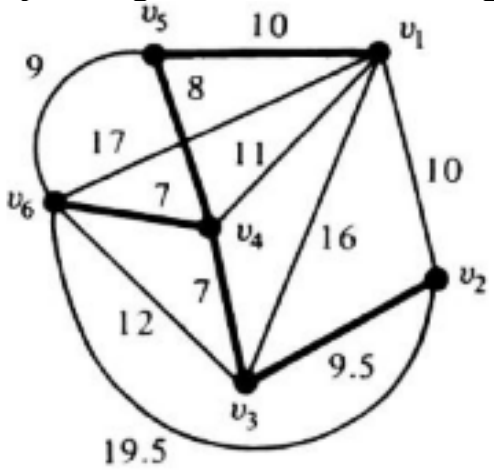
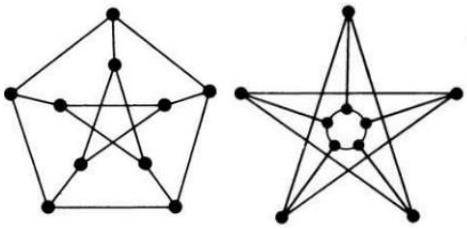


Fig. 1 *[(CO1)(Generate/HOCQ)]*

6 + 6 = 12

3. (a) Explain with relevant argument whether the graphs [Fig. 2(a) & (b) both] are Isomorphic.



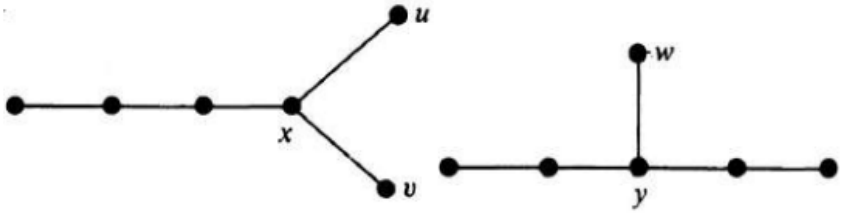


Fig. 2 (a) ***(b)*** *[(CO1)(Analyse/IOCQ)]*

(b) Show that a simple connected graph with n vertices has a maximum of $\frac{n(n-1)}{2}$ edges. *[(CO1) (Execute/IOCQ)]*

6 + 6 = 12

Group - C

4. (a) Generate the eigen values and eigen vectors of the following matrix

$$A=\begin{pmatrix} 3 & 1 & -1 \\ 2 & 2 & -1 \\ 2 & 2 & 0 \end{pmatrix}$$

[(CO2) (Develop/HOCQ)]

(b) Show that the set of vectors {(2,1,1), (1,2,2), (1,1,1)} is linearly dependent in $\mathbb{R}^3$. *[(CO2)(Apply/IOCQ)]*

6 + 6 = 12

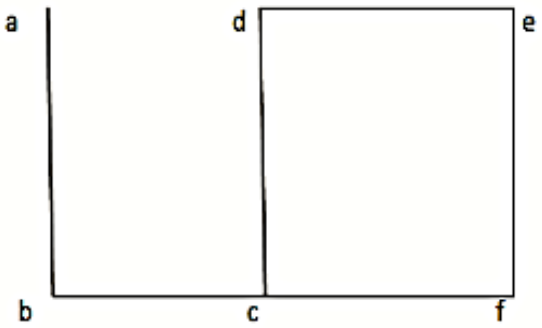
5. (a) Find the rank of the following matrix.

(i) $\begin{pmatrix} 0 & 0 & 2 & 2 & 0 \\ 1 & 3 & 2 & 4 & 1 \\ 2 & 6 & 2 & 6 & 2 \\ 3 & 9 & 1 & 10 & 6 \end{pmatrix}$ (ii) $\begin{pmatrix} 4 & 2 & 6 & -1 \\ 6 & 1 & 3 & 8 \\ 16 & 4 & 12 & 15 \\ 10 & 3 & 9 & 7 \end{pmatrix}$

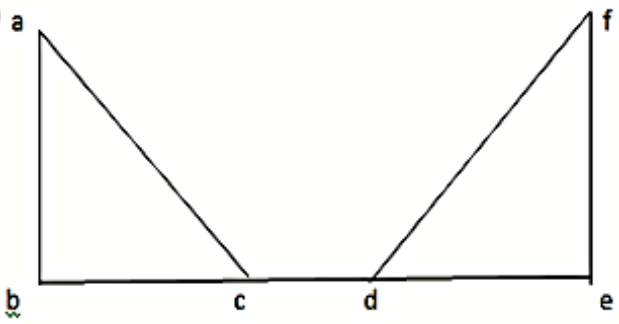
[(CO2)(Evaluate/HOCQ)]

(b) What do you mean by cut set, cut vertices and cut edges of a connected graph G. Find the cut vertices and cut edges for the following graphs.

(i)



(ii)



[(CO2)(Apply/IOCQ)]

(3 + 3) + (3 + 3) = 12

Group - D

6. (a) Three boxes of the same occurrence have the following proportion of black and white balls: Box 1 [5 black & 3 white], Box 2 [6 black & 2 white], Box 3 [3 black & 5 white]. One of the boxes is selected at random and one ball is randomly selected from it. What is the probability that the ball is black? *[(CO4)(Apply/IOCQ)]*

(b) Prove that if two events are independent then their complements are also independent. *[(CO4)(Remember/LOCQ)]*

(c) Each coefficients of the equation $ax^2 + bx + c = 0$ are determined by an ordinary die. What is the probability that the equation has real roots? *[(CO4)(Evaluate/HOCQ)]*

4 + 2 + 6 = 12

7. (a) If the regression equation of Y on X be $y = 0.57x + 6.93$ and X on Y be $x = 1.12y - 2.46$ Find the correlation coefficient between X and Y.

[(CO3)(Remember/LOCQ)]

(b) For the variables X and Y the equations of two regression lines are $4x - 5y + 33 = 0$ and $20x - 9y = 107$. Identify the regression line of Y on X.

[(CO3)(Understand/LOCQ)]

(c) Given $r_{xy} = 0.8$, $\Sigma xy = 60$, $\sigma_y = 2.5$ and $\Sigma x^2 = 90$. Calculate the number of items n.

[(CO3)(Analyse/IOCQ)]

4 + 4 + 4 = 12

Group - E

8. (a) If T_1 and T_2 are statistics with $E(T_1) = 2\theta_1 + 3\theta_2$ and $E(T_2) = \theta_1 + \theta_2$, then find the unbiased estimator of θ_1.

[(CO6)(Remember/LOCQ)]

(b) A random sample $x_1, x_2, \ldots, x_n$, is drawn from a Normal population with mean θ and variance 1. Prove that, $\frac{1}{n}\sum_{i=1}^{n}(x_i^2 - 1)$ is an unbiased estimator of θ^2.

[(CO6)(Remember/LOCQ)]

6 + 6 = 12

9. (a) Define the following, in context of hypothesis testing: null and alternative hypothesis, simple and composite hypothesis.

[(CO6)(Understand/LOCQ)]

(b) (i) If the sample observations are 2, 4, 6, 8 and 10 from an infinite population with variance σ^2, determine an unbiased estimate of σ^2. *[(CO6)(Understand/LOCQ)]*

(ii) Distinguish between the following, in context of hypothesis testing: test statistic, critical region.

[(CO6)(Understand/LOCQ)]

6 + (3 + 3) = 12

Cognition Level	LOCQ	IOCQ	HOCQ
Percentage distribution	35.42	39.58	25